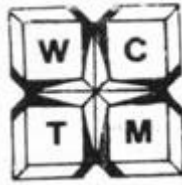


Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 1

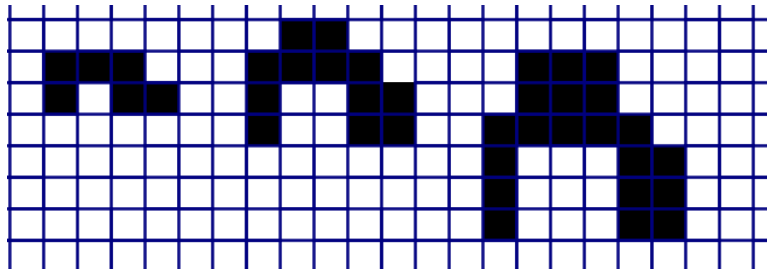
NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Consider the tiling pattern shown below. Find an expression for the number of tiles that would be expected in the 25th figure. Figures 1, 2 and 3 are shown.



- (A) 702 (B) 704 (C) 706 (D) 708 (E) 710 [1] _____

2. Half of the people in a room left. One third of those remaining started to dance. There were 12 people who were not dancing. The original number of people in the room was

- (A) 24 (B) 30 (C) 36 (D) 42 (E) 72 [2] _____

3. The scores on Test D and Test E are shown below. Describe the correlation between the scores on Test D and Test E.

Student	Test D	Test E
Buddy	303	20
Turner	343	53
Kathy	479	70
Suzanne	599	88

- (A) positive (B) negative (C) no correlation (D) quadratic (E) exponential [3] _____

4. Suppose that $\sqrt{\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \frac{6}{5} \cdot \dots \cdot \frac{a}{b}} = 3$ where $b = a - 1$. What is the value of $a + b$?

- (A) 11 (B) 17 (C) 18 (D) 35 (E) 37 [4] _____

5. A cube of edge 3 centimeters is cut into N smaller cubes, not all the same size. If the edge of each of the smaller cubes is a whole number of centimeters, then what is N ?

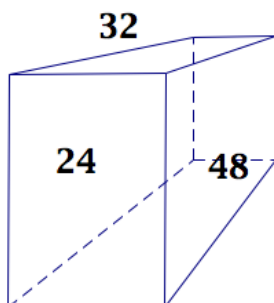
- (A) 4 (B) 8 (C) 12 (D) 16 (E) 20 [5] _____

Go to back \Rightarrow

6. What is the value of x if $4^{20} + 4^{20} = 2^x$?

- (A) 80 (B) 60 (C) 41 (D) 40 (E) 21 [6] _____
-

7. Given a rectangular box as shown below, find its volume in cubic centimeters knowing that the areas of the three different faces of the box are 24, 32 and 48 square centimeters.

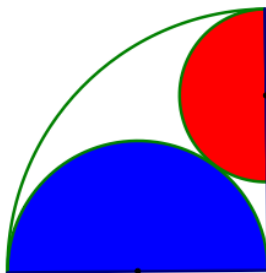


- (A) 104 (B) 192 (C) 208 (D) 576 (E) 36,864 [7] _____
-

8. A gambler sets up a game in which he puts four \$10 bills and one \$20 bill in one box. The person playing against the gambler blindly chooses one of the bills and keeps it. To break even on average, how much must the gambler charge this person to play?

- (A) \$16 (B) \$15 (C) \$14 (D) \$13 (E) \$12 [8] _____
-

9. In the figure below, the two semicircles are tangent. The radius of the quadrant and the diameter of the large semicircle are 2. Find the radius of the small semicircle.

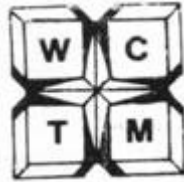


- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$ [9] _____
-

10. Which of the following represents an odd number in base 10?

- (A) 124_{five} (B) 134_{five} (C) 224_{five} (D) 334_{five} (E) 424_{five} [10] _____
-

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Test 2

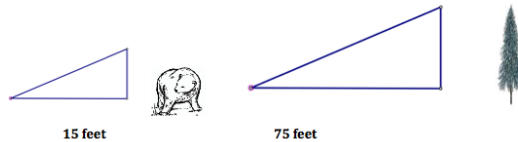
NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

11. A bear six feet tall casts a shadow fifteen feet long. At the same time, a nearby tree casts a shadow measuring seventy-five feet. How many feet tall is the tree?



- (A) 182.5 (B) 75 (C) 66 (D) 60 (E) 30 [11] _____

12. The following is a partial table of logarithms in the integer base a . Use these values to approximate $\log_a \frac{10}{3}$.

n	2	3	5	7
$\log_a n$	0.387	0.613	0.898	1.086

- (A) 0.523 (B) 0.672 (C) 0.694 (D) 1.086 (E) 1.204 [12] _____

13. The circumference of the base of a right-circular cylinder can is 12 inches. The height is 5 inches. A spiral stripe is painted on the can; it winds around exactly once as it reaches from the bottom to the top. The stripe reaches the top directly above the spot where it left the bottom. If the stripe is the shortest possible going from corner to corner, what is its length, in inches?

- (A) $5\sqrt{2}$ (B) $\sqrt{61}$ (C) 12 (D) 13 (E) 17 [13] _____

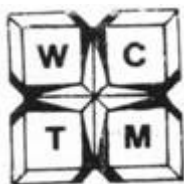
14. Four men, one of whom committed a crime, each made statements. If only one statement is true, who is the guilty man?

Painter	Chef	Fireman	Policeman
"The Chef did it."	"The policeman did it."	"I didn't do it."	"The Chef lied when he said I did it."

- (A) Chef (B) Policeman (C) Fireman (D) Painter (E) insufficient information [14] _____

Go to back \Rightarrow

Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 3

NAME: _____

CLASS 11 & 12 Grade

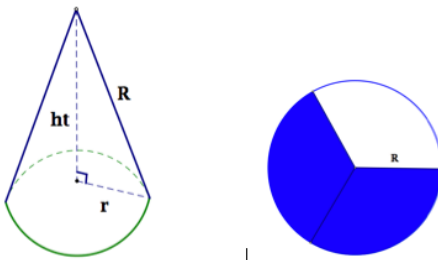
SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

21. A safety regulation states that the maximum angle of elevation for a rescue ladder is 72 degrees. If the fire department's longest ladder is 110 feet, what is the approximate maximum safe rescue height, in feet?

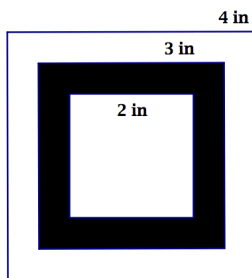
- (A) 27.9 (B) 34.0 (C) 72.0 (D) 104.6 (E) 106.4 [21] _____

22. A circular sheet of paper of radius 6 cm is cut into three equal sectors, and one sector is formed into a cone with no overlap. What is the height, in centimeters, of the cone?



- (A) $4\sqrt{3}$ (B) $4\sqrt{2}$ (C) $2\sqrt{6}$ (D) $2\sqrt{3}$ (E) 2 [22] _____

23. A dart is thrown randomly at the square target shown below. It is made from three concentric squares with side lengths 4 inches, 3 inches and 2 inches. What is the probability that it will hit the shaded region?



- (A) $\frac{1}{3}$ (B) $\frac{3}{4}$ (C) $\frac{5}{11}$ (D) $\frac{5}{16}$ (E) $\frac{9}{16}$ [23] _____

24. If a and b are integers such that $x^2 - x - 1$ is a factor of $ax^3 + bx^2 + 1$, then b is

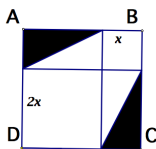
- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2 [24] _____

Go to back \Rightarrow

25. If $(5, 7)$ is a point on the graph of $y = f(x)$, find the coordinates of the corresponding point on the graph of $y = 3 \cdot f(2x + 7) - 5$.

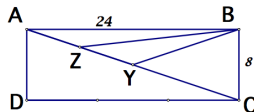
- (A) $(-1, 16)$ (B) $(3, 16)$ (C) $(17, 16)$ (D) $(3, 26)$ (E) $(5, 46)$ [25] _____

26. If $ABCD$ is a unit square, find the area of the shaded region, in square units.



- (A) $\frac{2}{9}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{7}{9}$ [26] _____

27. In rectangle $ABCD$, Y is the midpoint of \overline{AC} , Z is the midpoint of \overline{AY} , $AB = 24$ and $BC = 8$. Find the area of $\triangle ZBY$.

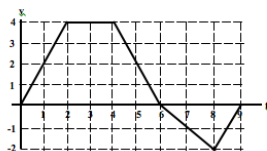


- (A) 12 (B) 16 (C) 18 (D) 24 (E) 32 [27] _____

28. For what value of k will $f(13) = 2009$ if $f(n) = \begin{cases} k & \text{if } n = 0 \\ f(n-1) + 20n & \text{if } n > 1 \end{cases}$?

- (A) 91 (B) 172 (C) 189 (D) 1820 (E) 1918 [28] _____

29. A bug begins to crawl up a vertical wire at time $t = 0$. The velocity of the bug at time t , $0 \leq t \leq 9$, is given by the function whose graph is shown below. At what time t does the bug change direction?

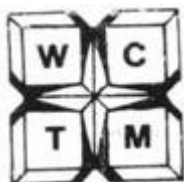


- (A) 2 (B) 4 (C) 6 (D) 8 (E) 9 [29] _____

30. What is the units digit of the decimal representation of 3^{2009} ?

- (A) 1 (B) 3 (C) 6 (D) 7 (E) 9 [30] _____

Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 4

NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

31. A basketball player is on the line to shoot a one-and-one free throw: with such a shot, the player may shoot a second basket only if the first shot is successful. If the player's free throw shooting average is $\frac{3}{4}$ what is the probability that she will score exactly one basket?

- (A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{3}{8}$ (D) $\frac{3}{16}$ (E) 1 [31] _____

32. If $2x + y = 10$, $x + |y| - y = 12$, and x is positive, then find the sum $x + y$.

- (A) -2 (B) 2 (C) $\frac{18}{5}$ (D) $\frac{22}{3}$ (E) 22 [32] _____

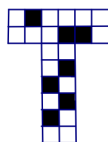
33. Evaluate $1^2 - 2^2 + 3^2 - 4^2 + \dots + 199^2$.

- (A) 21,000 (B) 19,900 (C) -19,700 (D) -19,900 (E) -21,000 [33] _____

34. Positive integers $A, B,$ and $C,$ with no common factor greater than 1, exist such that $A \log_{200} 5 + B \log_{200} 2 = C$. What is the value of $A + B + C$?

- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10 [34] _____

35. Match the pattern below with its folded figure.

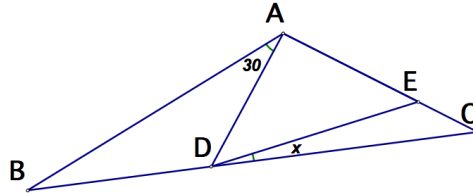


- (A) (B) (C) (D) (E) [35] _____

36. Let $f(x) = \sec x$ and $g(x) = \cot x$. Then find $g\left(f^{-1}\left(\frac{13}{5}\right)\right)$.

- (A) $\frac{5}{12}$ (B) $\frac{5}{18}$ (C) $\frac{5}{\sqrt{194}}$ (D) $\frac{12}{5}$ (E) $\frac{13}{12}$ [36] _____
-

37. In the figure below, $AB = AC$, $m\angle BAD = 30^\circ$ and $AE = AD$. What does x equal?



- (A) 5° (B) 10° (C) 15° (D) 20° (E) 30° [37] _____
-

38. In how many ways can the sum of \$2.40 be made up of nickels, dimes and quarters?

- (A) 81 (B) 110 (C) 135 (D) 158 (E) 162 [38] _____
-

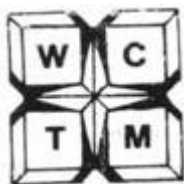
39. For which value of a could one use L'Hopital's Rule to evaluate the limit $\lim_{t \rightarrow a} \frac{t^3 - 4t}{t^2 - t - 6}$?

- (A) -2 (B) 0 (C) 2 (D) 3 (E) 6 [39] _____
-

40. Wanda weighed her four pet potbellied piglets in pairs. Together Oinker and Squeaker weighed 110 pounds, Squeaker and Curly weighed 103 pounds, and Curly and Porker totaled 108 pounds. How many pounds would Oinker and Porker weigh together?

- (A) 107 (B) 109 (C) 111 (D) 113 (E) 115 [40] _____
-

Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 5

NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

41. Which measure of center is best for a data set with outliers?

- (A) mean (B) median (C) range (D) interquartile range (E) mode [41] _____

42. A bag contains five different marbles. Each of three different students chooses one marble from the bag and then replaces it. Find the probability that all three students choose the same marble.

- (A) 0 (B) $\frac{1}{125}$ (C) $\frac{1}{25}$ (D) $\frac{3}{25}$ (E) $\frac{12}{25}$ [42] _____

43. A car accelerates from 0 mph to 20 mph in 3 seconds with constant acceleration. If it continues on to 60 mph with the same constant acceleration, how many feet will the car have traveled from the start until it reaches the speed of 60 mph? [Hint: 20 mph = $\frac{88}{3}$ feet per second and 60 mph = 88 feet per second.]

- (A) 88 (B) 264 (C) 396 (D) 528 (E) 792 [43] _____

44. Find the area of the region enclosed by the graph of $|y| + |2x| = 6$.

- (A) 12 (B) 18 (C) 36 (D) 48 (E) 72 [44] _____

45. If 92 percent of the members of the House of Representatives have stomach trouble, 90 percent have weak eyes, and 86 percent have liver ailments, what is the lowest percentage of members plagued with all three ailments?

- (A) 0 (B) 44 (C) 56 (D) 68 (E) 86 [45] _____

46. Factor $x^4 + 4$ as the product $(ax^2 + bx + c)(dx^2 + ex + f)$ with integral coefficients. Then $b \cdot e =$

- (A) -4 (B) -2 (C) 0 (D) 2 (E) 4 [46] _____

Go to back \Rightarrow

