Wolsborn-Drazovich STATE MATHEMATICS 52nd CONTEST, 2008

Test 1 **NAME**: _____

CLASS 11 & 12 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. Jeremy travels from point A to point B at 2 minutes per mile and returns over the same route at 2 miles per minute. Find his average speed, in miles per hour, for the whole trip.

   (A) 30 mph (B) 48 mph (C) 60 mph (D) 75 mph (E) 120 mph [1] _____

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2. A merchant pays an importation tax on certain goods at three different places. At the first he gives one-third of the goods' value; at the second he gives one-fourth of the value of the remainder. At the third he gives one-fifth the value of the remainder. The total tax is \$24. What is the original value of the goods?

   (A) \$30.64 (B) \$40.00 (C) \$60.00 (D) \$110.77 (E) \$144.00 [2] _____

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3. Imagine the continuation of the lattice below. What number would be **directly below** 100 in the continuation?

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 2 & 3 & 4 & & & \\ & & 5 & 6 & 7 & 8 & 9 & & \\ 10 & 11 & 12 & 13 & 14 & 15 & 16 & & \end{array}$$

   (A) 101 (B) 111 (C) 120 (D) 121 (E) 144 [3] _____

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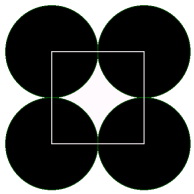
4. Four circles are arranged as shown below. Their centers are the vertices of a square drawn in white. If each circle has radius Q , what is the area of the unshaded region inside that square.



   (A) $Q^2\left(4-\frac{\pi}{4}\right)$ (B) $Q^2(1-\pi)$ (C) $Q^2(2-\pi)$ (D) $Q^2(4-\pi)$ (E) $4Q^2(1-\pi)$ [4] _____

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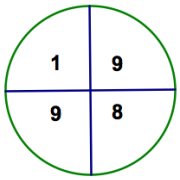
5. Find the product of all the real solutions of the exponential equation $16^{x^2+x+4} = 32^{x^2+2x}$.

   (A) 4 (B) $\frac{4}{3}$ (C) -4 (D) -12 (E) -16 [5] _____

Go to back⟹

6. The circular target shown below is divided into four congruent sectors with point values 1, 9, 9, and 8. Four darts are thrown at random and each scores. What is the probability that the digits obtained can be arranged as1998?



(A) $\frac{1}{64}$ (B) $\frac{3}{64}$ (C) $\frac{3}{32}$ (D) $\frac{3}{256}$ (E) $\frac{3}{16}$ [6] _____

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7. How many zeros does $f(x) = \cos(\log x)$ have on the interval $0 < x < 1$?

(A) 1 (B) 2 (C) 3 (D) infinitely many (E) none [7] _____

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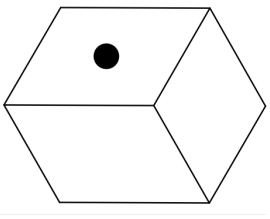
8. In a cube with an edge length of 10 units, a hole with a diameter of 2 units is drilled completely through, perpendicular to the base. Find the surface area of the resulting solid.



(A) $400 + 18\pi$ (B) $480 + 18\pi$ (C) $400 + 22\pi$ (D) $600 + 18\pi$ (E) $600 + 20\pi$ [8] _____

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9. Four numbers are written in a row. The mean of the first two numbers is 7, the mean of the middle two numbers is 2.3, and the mean of the last two numbers is 8.4. What is the mean of the first and last number?

(A) 5.9 (B) 7.7 (C) 8.9 (D) 11.4 (E) 13.1 [9] _____

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10. We interviewed forty-eight students about recycling paper (P), bottles (B), and cans (C). The following chart show the number of students who do **not** recycle one, or a combination of, these items. For example, seven students recycle only bottles according to the chart. How many recycle all three items?

P	B	C	PB	PC	BC	PBC
13	6	9	3	7	4	2

(A) 6 (B) 16 (C) 32 (D) 44 (E) 46 [10] _____

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Test 2 **NAME**: _____

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11. The sum of the first five terms of an arithmetic sequence is 40, and the sum of the first ten terms of the sequence is 155. Find the formula for the nth term.

(A) $2n+3$ (B) $3n+1$ (C) $3n+2$ (D) $3n-1$ (E) $3n$ [11] _____

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12. A game is played with two standard dice so that player A wins if the sum of 7 is rolled and player B wins if the sum of 4 or 10 is rolled. No one wins otherwise. What is the probability that B wins?

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$ (E) $\frac{1}{2}$ [12] _____

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13. The area of a rectangle is 360 square meters. If its length is increased by 10 meters and its width is decreased by 6 meters, then its area does not change. Find the perimeter of the original rectangle.

(A) 76 (B) 78 (C) 84 (D) 92 (E) 98 [13] _____

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14. The point $(4,3)$ is reflected about the horizontal x-axis to a point P. The P is reflected about the vertical y-axis to point Q. What is the sum of the coordinates of Q?

(A) 7 (B) 1 (C) 0 (D) -1 (E) -7 [14] _____

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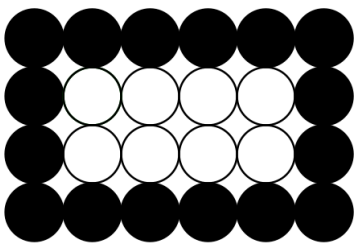
15. Consider a rectangular array of white poker chips. Place dark poker chips side-by-side around the white chips as illustrated below. Imagine an array for which the number of white chips equals the dark ones. In how many different ways can this happen? Do not count twice arrays that have a different orientation but have the same shapes. For example, don't count both a 3 by 5 and a 5 by 3 rectangle.



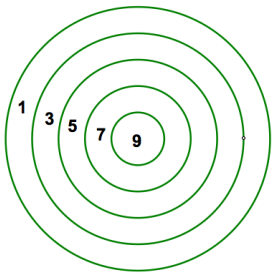
(A) none (B) 1 (C) 2 (D) 3 (E) infinitely many [15] _____

Go to back⟹

16. Two missiles speed directly toward each other, one at 9000 miles per hour and one at 21,000 miles per hour. If they start at 1317 miles apart, how far apart are they, in miles, one minute before they collide?

(A) 817 (B) 500 (C) 263 (D) 200 (E) 2.6 [16] _____

17. Luisa was playing darts. She threw six darts, and all hit the target shown below. The numbers 1, 3, 5, and 7 on the target are the scores awarded when a dart hits the ring, and 9 is the score for the center region. Which one of the following might have been her score?



(A) 17 (B) 28 (C) 29 (D) 31 (E) 56 [17] _____

18. A whale's head is 72 inches long; its tail is as long as its head plus half the length of the body, and its body is half its entire length. How long is the whale, in inches?

(A) 48 (B) 288 (C) 504 (D) 576 (E) 1152 [18] _____

19. The front wheel of Farmer Sprout's tractor has a radius of 25 cm and rotates at $2\frac{1}{2}$ revolutions per second. The back wheel has a diameter of 125 cm. At how many revolutions per second does it turn?

(A) 0.5 (B) 1 (C) 2 (D) 5 (E) 6.25 [19] _____

20. A particle is moving along the y-axis such that its y-coordinate at time t is given by the function $y(t) = t^3 - 3t^2 - 45t$. At time $t = 0$ which of the following statements is true?

(A) The particle is standing still.
(B) The particle is moving upward and gaining speed upward .
(C) The particle is moving upwards and losing speed upward.
(D) The particle is moving downward and gaining speed downward .
(E) The particle is moving downward and losing speed downward. [20] _____

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21. Two numbers have a product of 19,551 and a sum of 280. Find their positive difference.

(A) 14 (B) 133 (C) 196 (D) 280 (E) 19271 [21] _____

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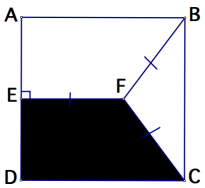
22. Figure $ABCD$ is a unit square . Find the area of the shaded region. Segments $\overline{EF}$, $\overline{BF}$ and $\overline{CF}$ all have the same length.



(A) $\frac{1}{3}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{11}{32}$ (E) $\frac{13}{32}$ [22] _____

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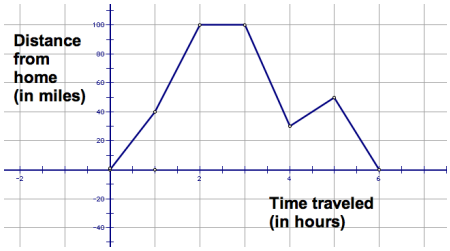
23. The graph below describes a six-hour trip that Ramon recently took, starting from his home. What was his average speed during that trip?



(A) 30 mph (B) 40 mph (C) 50 mph (D) 240 mph (E) 300 mph [23] _____

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24. What is the sum of all two-digit whole numbers whose squares end with the digits 21?

(A) 111 (B) 139 (C) 161 (D) 189 (E) 200 [24] _____

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25. A data set of five **unequal** numbers has range 6. Both the mean and the median of the set are 7. What is the product of the lowest and highest numbers in the set?

(A) 16 (B) 27 (C) 40 (D) 42 (E) 55 [25] _____

Go to back⟹

26. The following four hockey teams have each played each other once. If Montreal defeated Boston with the score 3-0, what was the score of the game between Toronto and Boston?

Team	Games Played	Won	Lost	Tied	Goals *For*	Goals Against
Montreal	3	3	0	0	7	0
Boston	3	1	1	1	2	3
Toronto	3	1	1	1	3	3
New York	3	0	3	0	1	6

(A) tie 3-3 (B) tie 1-1 (C) tie 0-0

(D) Toronto beats Boston 3-0 (E) Boston beats Toronto 2-0 [26] _____

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27. Wichita, Kansas is due north of Fort Worth, Texas. They lie on a circle centered at the Earth's center with approximate radius 3950 miles. The latitude of Wichita is approximately 37 degrees north, and Fort Worth is approximately 32 degrees north. What is the approximate distance, in miles, between the two cities?

(A) 172 (B) 345 (C) 688 (D) 1376 (E) 2206 [27] _____

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28. While shopping, a man purchased some scratch cards and won. His winnings equaled 20% of what he had when he left home. To celebrate, he went for dinner and spent 25% of everything he now had, leaving $180 in his wallet. How much did he spend for the dinner?

(A) $200 (B) $180 (C) $60 (D) $50 (E) $10 [28] _____

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29. Give the best approximation, in feet, for the smallest length for a diagonal of a rectangle with a perimeter of ten feet.

(A) 3.5 (B) 3.6 (C) 3.8 (D) 4.1 (E) 5.0 [29] _____

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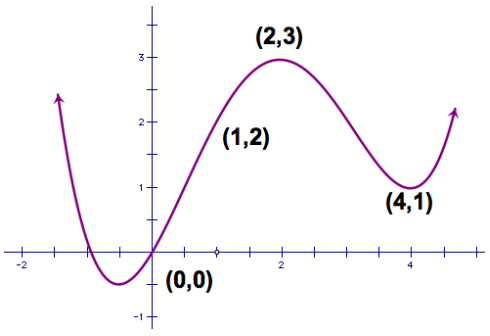
30. Based on the information in the graph, which expression has the **smallest** value?



(A) $\dfrac{f(4)-f(0)}{4}$ (B) $f'(1)$ (C) $f(2)$ (D) $f''(2)$ (E) $f''(4)$ [30] _____

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Test 4 **NAME**: _____

CLASS 11 & 12 Grade **SCHOOL**: _____

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31. The supplement of an angle is 78° less than twice the supplement of the complement angle. Find measure of the angle.

(A) 26° (B) 52° (C) 78° (D) 102° (E) 174° [31] _____

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32. 72 identical banquet dinners cost a total of \$__67.9__ . What is the sum of the two missing digits?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9 [32] _____

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33. If $\log_b(xy) = 11$ and $\log_b\left(\frac{x}{y}\right) = 5$, what is $\log_b(x)$?

(A) 3 (B) 8 (C) 16 (D) 32 (E) 55 [33] _____

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34. Marley's age is 16 more than the sum of Sam's age and Amber's age. The square of Marley's age is 1632 more than the square of the sum of Sam's age and Amber's age. What is the sum of Marley's, Sam's and Amber's ages?

(A) 51 (B) 59 (C) 67 (D) 102 (E) 118 [34] _____

---

35. Anna says that a 55% chance exists that she will go to the movie tomorrow if it is raining at noon, a 30% chance if it is not raining at noon. Willard forecasts a 40% chance of rain at noon. On the basis of these numbers, what is the probability that Anna will go to the movie?

(A) 15% (B) 22% (C) 40% (D) 45% (E) 95% [35] _____

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36. What is the sum of the digits of the decimal form of $2^{2006} \cdot 5^{2008}$?

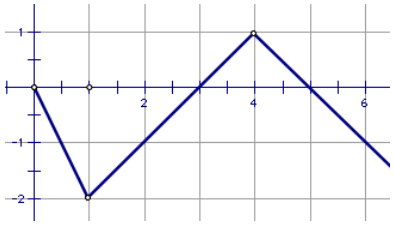
(A) 2 (B) 4 (C) 5 (D) 7 (E) 10 [36] _____

Go to back⟹

37. The "adderage" of two fractions is defined as the sum of their numerators over the sum of their denominators. When, if at all, will the adderage of two fractions equal the average of the same two fractions?

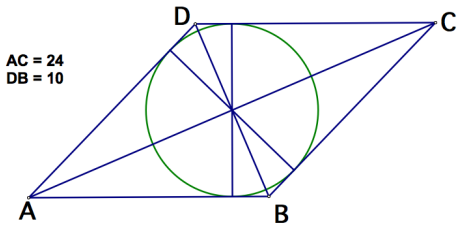
(A) They are never equal.
(B) They are equal when the numerators are equal.
(C) They are equal when the denominators are equal.
(D) They are equal when fractions are negative reciprocals.
(E) They are equal when the fractions are reciprocals. [37] _____

38. Given the graph of the function $f(x)$ below, find the value of $\int_1^5 f(x)\,dx$.



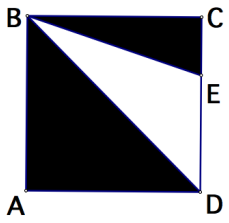
(A) -2 (B) -1 (C) 0 (D) 1 (E) 4 [38] _____

39. What is the radius of the circle inscribed in a rhombus with diagonals of length 10 and 24?



(A) 4 (B) $\frac{58}{13}$ (C) $\frac{60}{13}$ (D) 5 (E) 6 [39] _____

40. Square $ABCD$ has an area of 36 m^2. $DE = 2EC$. What is the ratio of the area of ΔBED to the area of square $ABCD$?



(A) $\frac{5}{18}$ (B) $\frac{11}{36}$ (C) $\frac{1}{3}$ (D) $\frac{13}{36}$ (E) $\frac{7}{18}$ [40] _____

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Test 5 **NAME**: _____

CLASS 11 & 12 Grade **SCHOOL**: _____

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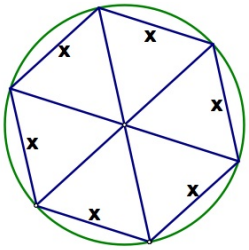
41. Grovel the gardener had a circular garden with a regular hexagonal lawn inscribed in it. The lawn was made from $\sqrt{108}$ square meters of turf. What was the approximate diameter of the garden?



(A) 1.8 (B) 2 (C) 2.8 (D) 4 (E) 4.9 [41] _____

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42. Determine the maximum value of $x+y+z$ given $x^2+y^2+z^2 = xy+xz+yz = 3$.

(A) $3\sqrt{3}$ (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{3}$ (D) 3 (E) $\frac{1}{3}$ [42] _____

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43. There are four stages in life: child, youth, young adult and mature adult. Democratius has lived $\frac{1}{6}$ of his life as a child, $\frac{1}{8}$ of his life as a youth, $\frac{1}{2}$ of his life as a young adult, and has spent 15 years as a mature adult. How old is Democratius?

(A) 96 (B) 72 (C) 60 (D) 48 (E) 24 [43] _____

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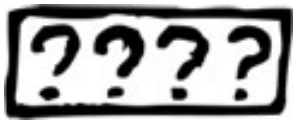
44. A box contains exactly five chips, three of which are red and two of which are white. Chips are randomly removed one at a time without replacement until all red chips are drawn or all white chips are drawn. What is the probability that the last chip drawn is white?



(A) $\frac{1}{10}$ (B) $\frac{6}{25}$ (C) $\frac{2}{5}$ (D) $\frac{50}{92}$ (E) $\frac{3}{5}$ [44] _____

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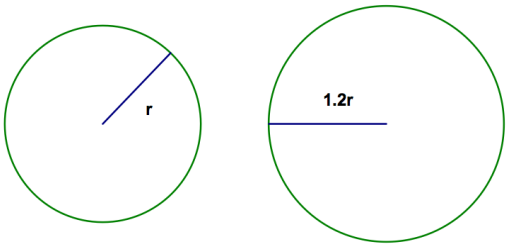
45. Find the sum of all real x so that $\sqrt{1-\sqrt{1-x}} = x$

(A) $\sqrt{5}$ (B) 1 (C) 0 (D) -1 (E) $-1+\sqrt{5}$ [45] _____

Go to back⟹

46. If the radius of a circle is increased by 20%, by what percent is the area increased?



(A) 20% (B) 40% (C) 44% (D) 140% (E) 144% [46] _____

47. Select the equation solved by the value x represented below.

$$x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$$

(A) $x^2 = 6$ (B) $x^2 = 6 + \sqrt{6}$ (C) $x^2 = 6 + x$ (D) $x^2 = 36 + x$ (E) $x^2 = \sqrt{6}\,x$ [47] _____

48. How many values of x can be found so that $0 \le x \le 2\pi$ and $\sin x + \cos x = 1$?

(A) none (B) one (C) two (D) three (E) four [48] _____

49. Find a value of a that will make f a continuous function given: $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 2x}{x - 1} & \text{if } x < 1 \\ 2x + a & \text{if } x \ge 1 \end{cases}$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2 [49] _____

50. Find the sum of all even three-digit positive integers that are multiples of 3.

(A) 81,801 (B) 82,350 (C) 163,053 (D) 164,700 (E) 182,817 [50] _____

Grades 11-12
2008 Math Contest Exam Answer Key

Exam	T1	T2	T3	T4	T5
P1	b	d	a	a	d
P2	b	a	e	a	d
P3	c	a	b	b	b
P4	d	e	e	d	e
P5	e	c	c	c	c
P6	e	b	c	d	c
P7	d	b	b	c	c
P8	d	d	c	b	d
P9	e	b	a	c	d
P10	c	d	d	c	b