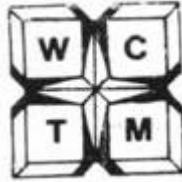


Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007



Test 1

NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. If P represents the product of all prime numbers less than 1000, what is the value of the units digit of P ?

- (A) 0 (B) 1 (C) 2 (D) 6 (E) 8 [1] _____
-

2. If it is 11 A.M. right now, what time will it be 2007 hours from now?

- (A) 12:00 AM (B) 12:00 PM (C) 1:00 AM (D) 2:00 AM (E) 3:00 AM [2] _____
-

3. Triangle ABC is equilateral, and \overline{AD} is an altitude. If the length of \overline{AB} is 12, find the distance from B to the midpoint of \overline{AD} .

- (A) 3 (B) 6 (C) $3\sqrt{3}$ (D) $3\sqrt{5}$ (E) $3\sqrt{7}$ [3] _____
-

4. How many triangles are in the figure below?

- (A) 7 (B) 18 (C) 23 (D) 35 (E) 43 [4] _____
-

5. The mean of a set of numbers is 120. If one number in the set is increased by 300, the mean increases to 135. How many numbers are in the set?

- (A) 12 (B) 15 (C) 18 (D) 20 (E) 24 [5] _____
-

Go to back \implies

6. How many solutions does the equation $x = 4 \cos(x)$ have if $-5 \leq x \leq 5$?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4 [6] _____
-

7. What is the units digit in the decimal representation of 7^{2007} ?

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9 [7] _____
-

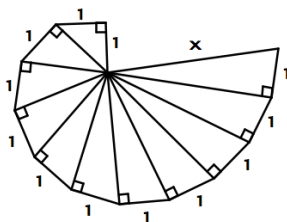
8. George had a number of jelly beans and ate one-third of them. Martha found the remaining jelly beans and ate some until only one-fifth of the original number of jelly beans remained. What fraction of the jelly beans that Martha found did she eat?

- (A) $\frac{2}{3}$ (B) $\frac{7}{15}$ (C) $\frac{3}{5}$ (D) $\frac{7}{10}$ (E) $\frac{3}{10}$ [8] _____
-

9. Suppose that $4x - 9 \leq f(x) \leq x^2 - 4x + 7$ for $x \geq 0$. Find $\lim_{x \rightarrow 4} f(x) =$

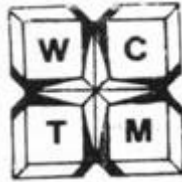
- (A) -9 (B) 0 (C) 1 (D) 7 (E) doesn't exist [9] _____
-

10. Determine the length of the segment labeled x in the figure below.



- (A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{10}$ (D) $2\sqrt{2}$ (E) $3\sqrt{3}$ [10] _____
-

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007



Test 2

NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. What number is doubled when two-thirds of it is added to 36?

- (A) 18 (B) 24 (C) 27 (D) 30 (E) 36 [1] _____
-

2. A boy is 160 centimeters tall. If he could walk around the earth along the equator, the top of his head would travel further than his feet. Estimate how much further (in meters) his head would travel. Treat the Earth as a sphere with radius 6378 kilometers.

- (A) 100 (B) 10 (C) 400 (D) 4000 (E) 40,000 [2] _____
-

3. The sum of five consecutive integers is 5^3 . Find the product of the smallest and largest of them.

- (A) 500 (B) 616 (C) 621 (D) 625 (E) 750 [3] _____
-

4. A bicycle wheel has a 12 inch radius. Approximately how many revolutions are required to travel five miles on this bicycle?



- (A) 840 (B) 2100 (C) 3600 (D) 4200 (E) 8400 [4] _____
-

5. A gambler has two coins in his pocket – one fair coin and one two-headed coin. He selects at random one of these coins, and then flips the coin twice. If he gets two heads, what is the probability that he selected the fair coin?

- (A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$ [5] _____
-

6. Find the sum of the coefficients in the expansion of $(a + b + c)^8$.

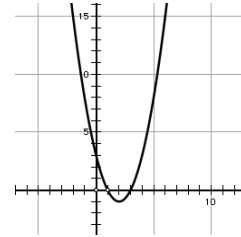
- (A) 3 (B) 24 (C) 27 (D) 256 (E) 6561 [6] _____
-

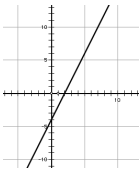
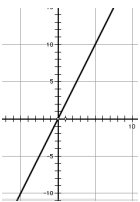
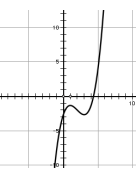
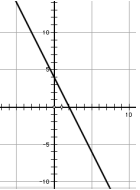
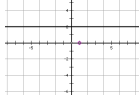
Go to back \Rightarrow

7. Of ten boxes, five contain pencils, four contain pens, and two contain both pens and pencils. How many boxes contain neither pens nor pencils?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5 [7] _____

8. The function $f(x)$ is given in the sketch below. Select the sketch of $f'(x)$.

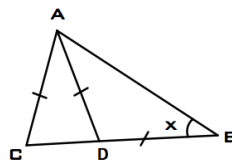


- (A)  (B)  (C)  (D)  (E)  [8] _____

9. Consider a circle with a chord 5 units from the center. If the length of that chord is 10 units, then how long is the diameter?

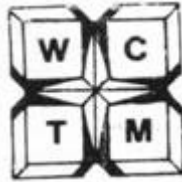
- (A) 10 (B) $10\sqrt{2}$ (C) 15 (D) 5 (E) $5\sqrt{5}$ [9] _____

10. In the triangle below, $\overline{AB} = \overline{CB}$ and $\overline{AC} = \overline{AD} = \overline{DB}$. Determine the measure of the angle x .



- (A) 18° (B) 30° (C) 36° (D) 64° (E) 72° [10] _____

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007



Test 3

NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The sum of the second and the fifth terms of an arithmetic sequence is equal to the sixth term of the sequence. If the sixth term is 25, find the tenth term of the sequence.

(A) 30 (B) 40 (C) 45 (D) 50 (E) 55 [1] _____

2. Let $f(x) = \sin(x)$ and let h be the smallest positive real number so that the graph of $y = f(x - h)$ will be symmetric about the y -axis. Find $\sin(h)$.

(A) 1 (B) -1 (C) 0 (D) $-\frac{2}{\sqrt{2}}$ (E) $\frac{2}{\sqrt{2}}$ [2] _____

3. Anna says that a fifty-five percent chance exists that she will go to the movie tomorrow if it is raining at noon, and a thirty percent chance she'll go if it is not raining. Willard forecasts a forty percent chance of rain at noon. On the basis of these numbers, what is the probability that Anna will go to a movie?

(A) 12% (B) 18% (C) 22% (D) 40% (E) 70% [3] _____

4. What is the maximum number of intersection points when two circles and three straight lines intersect each other? Assume that the figure is drawn in a plane and that no figure coincides with another.

(A) 12 (B) 15 (C) 16 (D) 17 (E) 18 [4] _____

5. If $\log_b(xy) = 11$ and $\log_b\left(\frac{x}{y}\right) = 5$, what is $\log_b x$?

(A) 6 (B) 8 (C) 16 (D) 32 (E) $\frac{11}{5}$ [5] _____

6. Consider the function $f(x)$ defined on the interval $[0, 1]$ with all of the following properties:

(a) $f(x)$ is continuous on $[0, 1]$.

(b) $f(x)$ is decreasing on the entire interval $[0, 1]$.

(c) $f(0.1) = 1, f(0.3) = 0.5, f(0.7) = -0.1$ and $f(0.9) = -0.5$.

According to the Intermediate Value Theorem, which one value might solve $f(x) = 0$?

(A) $x = 0.2$ (B) $x = 0.3$ (C) $x = 0.5$ (D) $x = 0.8$ (E) none of these [6] _____

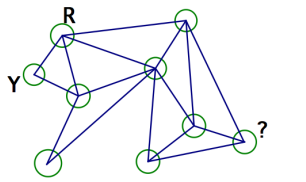
Go to back \Rightarrow

7. Daddy Warbucks was a cartoon character started in 1924. He was called a tycoon, and has come to symbolize people with lots of money. Suppose that Daddy Warbucks is very methodical, and knows just how many \$100, \$500 and \$1 bills he's carrying. The bigger denominations are for his impulse buys. He carries a specific number of \$1 for tips. If he has 500 bills total in his briefcase, and their total value is \$50,000, how many \$1 bills must he be carrying?



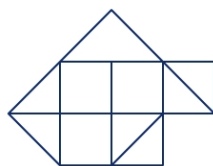
- (A) 100 (B) 200 (C) 300 (D) 400 (E) 500 [7] _____

8. Consider the graph below. Each circle at a vertex is colored red, yellow or blue. Once colored, no two vertices of a triangle are colored the same color. What is the color of the vertex marked with the question mark?



- (A) red (B) yellow (C) blue (D) red or yellow (E) blue or yellow [8] _____

9. It is impossible to draw the following design without lifting your pencil from the page, traversing each and every edge precisely once. However, if you are permitted to trace over a small number of edges twice in order to trace over the entire figure, what is the least number of reused edges needed?

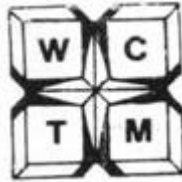


- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5 [9] _____

10. On a certain test, the average score for the women in the class is 83, whereas the average score for the men is 71. If the average score of all students is 80, what percent of the students are women?

- (A) 25% (B) $33\frac{1}{3}\%$ (C) 50% (D) $66\frac{2}{3}\%$ (E) 75% [10] _____

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007



Test 4

NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Every system of linear equations can be represented by an augmented matrix. For example, the 2×2 system
- $$\begin{cases} x + 2y = 6 \\ -2x + y = 5 \end{cases} \text{ can be represented as } \begin{bmatrix} 1 & 2 & 6 \\ -2 & 1 & 5 \end{bmatrix}.$$

For what value of k will $\begin{bmatrix} -3 & -5 & 36 & 2k \\ -1 & 0 & 7 & k \\ 1 & 1 & -2k & -4 \end{bmatrix}$ represent a system with infinitely many solutions?

- (A) -2 (B) 0 (C) 3 (D) 5 (E) 6 [1] _____

2. Which of the following equals a rational number?

- (A) $\log_5 19 - \log_5 9$ (B) $\log_5 19 - \log_5 95$ (C) $\log_5 (199^5)$
 (D) $\log_5 |10 - 5\sqrt{5}|$ (E) $\frac{\log_5 100}{\log_5 4}$ [2] _____

3. Consider the following statements. How many statements are true for all values of x ?

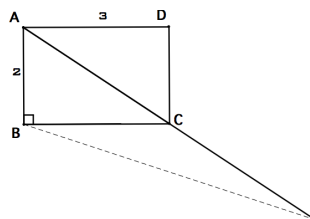
$$\begin{array}{lll} \tan x \cot x = 1 & \sin^2(6x) + \cos^2(6x) = 1 & \sin(2x) = 2 \sin x \cos x \\ \cos(-x) = -\cos(x) & \cos(2x) = 2 \cos^2 x - 1 & \end{array}$$

- (A) none (B) one (C) two (D) three (E) four [3] _____

4. The integers 1 through 1000 are printed on a list. How many times does the digit 3 appear on the list?

- (A) 30 (B) 100 (C) 300 (D) 330 (E) 333 [4] _____

5. The diagonal of rectangle $ABCD$ is extended to E such that $\overline{AC} = \overline{CE}$. Find the length of segment \overline{BE} given $\overline{AD} = 3$ cm and $\overline{AB} = 2$ cm. Hint: It may be helpful to draw another triangle by extending \overline{AB} .



- (A) 6 (B) $\sqrt{13}$ (C) $2\sqrt{10}$ (D) $2\sqrt{13}$ (E) $\sqrt{22}$ [5] _____

Go to back \Rightarrow

6. There is a rule in base 10 to determine parity: the last digit reveals whether a number is even or odd. Now consider numbers written in base 7. Which **one** of the following is a legitimate rule in base 7 to identify an even (base 7) number?

- (A) The number is even when the units digit is even.
- (B) The number is even when the sum of the digits is even.
- (C) The number is even when the sum of the digits is odd.
- (D) The number is even when the product of its digits is odd.
- (E) The number is even when the product of its digits is even.


[6] _____

7. Suppose the f and g are two continuously differentiable, real-valued functions for which: $f(3) = 4$; $g(3) = 2$; $f'(3) = -6$; and $g'(3) = 5$. Use this information to evaluate $\left(\frac{f}{f-g}\right)'(3)$.

- (A) 2
- (B) 4
- (C) 6
- (D) 8
- (E) 10

[7] _____

8. Bob has trapped a mouse in a pipe 10 feet long. He has also placed a piece cheddar cheese at the left end and a piece of swiss cheese at the right end. The mouse is running back and forth inside the pipe. Its distance $D(t)$ from the cheddar cheese t seconds after he was trapped is approximately expressed by the function

$D(t) = 5 \sin(2t) + 5$. When $t = \frac{7\pi}{2}$, how fast is the mouse running and in what direction? 

- (A) 5 ft/sec toward cheddar
- (B) 5 ft/sec toward swiss
- (C) 10 ft/sec toward cheddar
- (D) 10 ft/sec toward swiss
- (E) He's stopped.

[8] _____

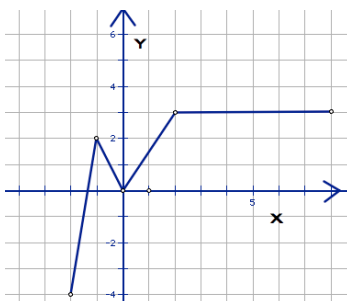
9. Three darts are thrown at a target shown below. Each lands in a different small square. What is the probability that the squares in which they land form a vertical, horizontal or diagonal row?



- (A) $\frac{2}{21}$
- (B) $\frac{3}{42}$
- (C) $\frac{1}{3}$
- (D) $\frac{1}{7}$
- (E) $\frac{1}{9}$

[9] _____

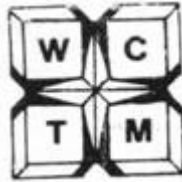
10. The function f is a piece-wise continuous real-valued function whose graph is given by the sketch below. For which values of x does $f(x+3) = f(x) + 1$?



- (A) $x = 1, -1$
- (B) $x = 1, \frac{4}{3}$
- (C) $x = -1, \frac{4}{3}$
- (D) $x = -\frac{4}{3}, \frac{4}{3}$
- (E) none

[10] _____

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007



Test 5

NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The roots of $x^3 + 4x^2 - 7x + a = 0$ are $-5, -1,$ and 2 . What is the sum of the roots of $(x - 3)^3 + 4(x - 3)^2 - 7(x - 3) + a = 0$?

(A) -13 (B) -4 (C) 0 (D) 2 (E) 5 [1] _____

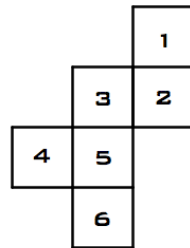
2. Consider complex number $a + bi$, where a and b are real numbers and $i^2 = -1$. $(a + bi)^2 = 2i$. What is the value of b ?

(A) 1 (B) 1 or -1 (C) $\sqrt{2}$ (D) $\sqrt{2}$ or $-\sqrt{2}$ (E) 2 [2] _____

3. Consider the set $S = \{1, x, y, x + y\}$ with four data points. Placing them in order, we have that $1 < x < y < (x + y)$. Determine the difference between the mean and the median of S .

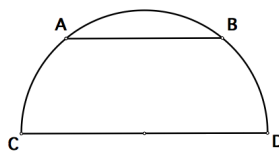
(A) $\frac{2x - 2y + 1}{4}$ (B) $\frac{2y - 2x + 1}{4}$ (C) $\frac{x + y + 1}{4}$ (D) $\frac{4y + 1}{4}$ (E) $\frac{1}{4}$ [3] _____

4. When the net of six squares is cut out and folded, it forms a cube. What is the product of the numbers on the four faces adjacent to the one labeled with a 1?



(A) 120 (B) 144 (C) 180 (D) 240 (E) 360 [4] _____

5. The circle shown below has radius 2 cm. The chord \overline{AB} is parallel to the diameter \overline{CD} . If \overline{AB} is half as long as the diameter, what is the distance from the chord to the diameter?



(A) 2 (B) 1 (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{1}{2}$ [5] _____

Go to back \implies

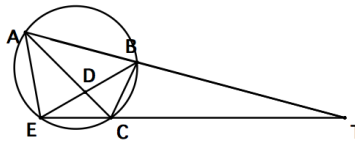
6. The following expression defines an operation \boxtimes .

$$a \boxtimes b = \frac{a+b}{a-b}$$

Use it to evaluate the expression $(6 \boxtimes 4) \boxtimes 3$.

- (A) 15 (B) 5 (C) 4 (D) 1 (E) $\frac{13}{7}$ [6] _____
-

7. In the figure below, $m\angle ATE = 15^\circ$, the measure of circular arc AB is 140° , and the measure of arc EC is 70° . Determine $m\angle ACE$.

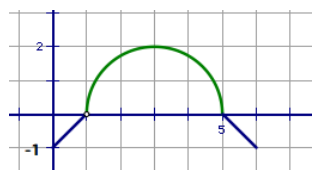


- (A) 22.5° (B) 35° (C) 45° (D) 67.5° (E) 70° [7] _____
-

8. In how many positive integers less than one million do each of the digits 1, 2, 3, 4, and 5 appear **exactly** once, in any order? For example, each of the three numbers 345,192, 14,235 and 530,412 qualify.

- (A) 5^7 (B) 5^6 (C) 5^5 (D) $5^2 \cdot 4^5$ (E) $5 \cdot 6 \cdot 5!$ [8] _____
-

9. The function $f(x)$ is defined by the graph below. Evaluate the integral $\int_0^6 f(x) dx$. Hint: you may assume that the curve over $[1, 4]$ is a semi-circle.



- (A) 4π (B) 2π (C) $2\pi + 1$ (D) $2\pi - 1$ (E) $4\pi + 1$ [9] _____
-

10. How many of the following statements about y , where $y = \tan^{-1}(\tan x)$, are false?

- It is undefined at $x = \pi + k\pi$ where $k = \pm 1, \pm 2, \dots$.
- It is undefined at $x = \frac{\pi}{2} + k\pi$ where $k = \pm 1, \pm 2, \dots$.
- It is defined for all real values of x .
- Its range equals the set of all reals.
- Its graph is symmetric about the origin.

- (A) one (B) two (C) three (D) four (E) five [10] _____
-

Grades 11-12
2007 Math Contest Exam Answer Key

Exam	T1	T2	T3	T4	T5
P1	a	c	c	d	e
P2	d	b	a	b	b
P3	e	c	d	d	e
P4	e	d	d	c	b
P5	d	c	b	c	c
P6	d	e	c	b	c
P7	b	c	d	d	c
P8	d	a	b	c	e
P9	d	b	b	a	d
P10	a	c	e	c	c