

Wolsborn-Drazovich STATE MATHEMATICS CONTEST, 2006
50th Anniversary



Test 1

NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Convert the base 8 number 146_8 to base 3.

- (a) 10210_3 (b) 1210_3 (c) 121_3 (d) 102_3 (e) 12_3 [1] _____
-

2. If $f'(a) = 0$ at a certain value a , which one statement concerning $f(a)$ is correct?

- (a) $(a, f(a))$ is a point at which the tangent line is parallel to the y -axis.
(b) $(a, f(a))$ is a point at which the tangent line is parallel to the x -axis.
(c) $(a, f(a))$ is a local maximum.
(d) $(a, f(a))$ is a local minimum.
(e) $(a, f(a))$ is a point of inflection. [2] _____
-

3. For what value of k is the following statement true?

$$\sqrt{\frac{a}{b} \sqrt{\frac{b}{a} \sqrt{\frac{a}{b}}}} = \left(\frac{a}{b}\right)^k$$

- (a) $-\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) $\frac{3}{8}$ (e) $-\frac{1}{2}$ [3] _____
-

4. Three identical balls are marked 1, 2, and 3 and placed in an urn. One ball is drawn, its number is recorded, and then the ball is returned to the urn. After repeating the process twice more, the sum of the numbers is recorded as 6. What is the probability that the ball numbered 2 was drawn in all three tries?

- (a) $\frac{1}{10}$ (b) $\frac{1}{9}$ (c) $\frac{1}{7}$ (d) $\frac{7}{9}$ (e) $\frac{7}{27}$ [4] _____
-

5. Describe the real numbers that comprise the domain of the real-valued function $f(x) = \sqrt{5 - \sqrt{x}}$.

- (a) $x \geq 0$ (b) $x \geq 25$ (c) $x \leq 25$ (d) $0 \leq x \leq \sqrt{5}$ (e) $0 \leq x \leq 25$ [5] _____
-

6. Which one of the following properties is true about the solutions of $2x^2 + 18 = 0$?

- (a) The product of the solutions is positive.
(b) The product of the solutions is zero.
(c) The product of the solutions is negative.
(d) The product of the solutions is an imaginary number.
(e) None of the above. [6] _____
-

Go to back \Rightarrow

7. A square has lines drawn in it that subdivide its interior. First, the diagonals are drawn. Lines are also drawn so that midpoints of the adjacent sides are connected. Determine the number of triangles contained in the figure.

- (a) 10 (b) 12 (c) 14 (d) 16 (e) 20 [7] _____
-

8. A set of cards is labeled with integers 1 to 200 and put into a bag. What is the probability that a card labeled with a number divisible by either 2 or 5 could be drawn?

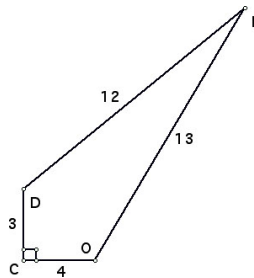
- (a) $\frac{1}{2}$ (b) $\frac{140}{200}$ (c) $\frac{120}{200}$ (d) $\frac{40}{200}$ (e) $\frac{20}{200}$ [8] _____
-

9. Although the Pacific Ocean, the Baltic Sea, and the Arctic Ocean are the only three bodies of water, they add up to the seven Cs. Substitute a different digit for each letter in the following statement. Note that the number 1 is not used. Which of the following statements about the letters used is **true**?

$$\begin{array}{r}
 P A C I F I C \\
 B A L T I C \\
 + A R C T I C \\
 \hline
 C C C C C C C
 \end{array}$$

- (a) $C = 5$ (b) $C = 9$ (c) $I = 7$ (d) $F = 8$ (e) $A = 4$ [9] _____
-

10. Find the area of the quadrilateral COLD drawn below.



- (a) 4 (b) 6 (c) 9 (d) 24 (e) 36 [10] _____
-

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Test 2

NAME: _____

CLASS 11 & 12 Grade

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SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Simplify the following expression so that each variable appears only once and none has a negative exponent:

$$\frac{a^{-5}b^{-9}c^7d^4e^{-2}}{b^{-14}c^{-3}d^7e^4f^{-3}}$$

- (a) $\frac{-a^5c^4}{b^5d^3e^2f^3}$ (b) $\frac{a^5c^4}{b^5d^3e^2f^3}$ (c) $\frac{b^5c^{10}f^3}{a^5d^3e^6}$ (d) $\frac{b^5c^4f^3}{a^5d^3e^2}$ (e) $\frac{-a^5b^5c^4}{d^3e^2f^3}$ [1] _____
-

2. Three dart players threw simultaneously at a tic-tac-toe board, each hitting a different square. What is the probability that the three hits constituted a win at tic-tac-toe?

- (a) $\frac{1}{3}$ (b) $\frac{4}{25}$ (c) $\frac{8}{27}$ (d) $\frac{2}{19}$ (e) $\frac{2}{21}$ [2] _____
-

3. Find the area of the region enclosed by the graph of $|y| + |2x| = 6$.

- (a) 24 sq units (b) 36 sq units (c) 48 sq units (d) 72 sq units (e) 18 sq units [3] _____
-

4. The 120 permutations of WDSMC are arranged in dictionary order, as if each were an ordinary five-letter word. Find the last letter of the 86th word in the list.

- (a) W (b) D (c) S (d) M (e) C [4] _____
-

5. Find the slope of the tangent line to the curve $f(x) = x^2 - 3x - 1$ at the point $\left(\frac{1}{2}, \frac{9}{4}\right)$.

- (a) -2 (b) -1 (c) 0 (d) 1 (e) 2 [5] _____
-

6. If $(4, 7)$ is a point on the graph of $y = f(x)$, which of the following points is the corresponding point on the graph of $g(x) = 3f(2(x + 3)) - 5$?

- (a) $(5, 16)$ (b) $(11, -3)$ (c) $(-1, -3)$ (d) $(-1, 16)$ (e) $(5, -3)$ [6] _____
-

Go to back \Rightarrow

7. Solve the inequality $mx + n < k$ for x . Assume that $m < 0$.

- (a) $x < \frac{k-n}{m}$ (b) $x > \frac{k-n}{m}$ (c) $x > \frac{n-k}{m}$ (d) $x > k - n - m$ (e) $x < k - n - m$ [7] _____
-

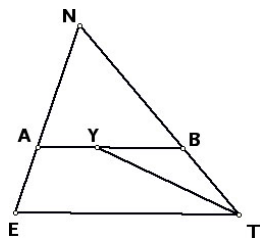
8. A culture of 100 bacteria doubles in number every two hours. How many hours will it take for the number of bacteria to reach 3200?

- (a) 32 (b) 16 (c) 12 (d) 10 (e) 8 [8] _____
-

9. Which one of the following functions **IS NOT** periodic with period 1 and amplitude 1?

- (a) $y = \cos(2\pi x - 1)$ (b) $y = \cos(2\pi x) + 3$ (c) $y = \sin(2\pi x - e) + 1$ [9] _____
(d) $y = \sin(2\pi x)$ (e) $y = \cos(2\pi x) + \sin(2\pi x)$
-

10. Given: $\overline{AB} \parallel \overline{ET}$; \overline{TY} bisects $\angle BTE$; $m\angle ANB = 60^\circ$; and $m\angle NAB = 70^\circ$. Find $m\angle BYT$.



- (a) 40° (b) 35° (c) 30° (d) 25° (e) 20° [10] _____
-

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Test 3

NAME: _____

CLASS 11 & 12 Grade

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SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The two diagonals of a rhombus measure 10 and 24. What is its perimeter?

- (a) 26 (b) 44 (c) 52 (d) 68 (e) 120 [1] _____
-

2. The slope of the line $y = 2$ is

- (a) 0 (b) -2 (c) 2 (d) 1 (e) Undefined [2] _____
-

3. A drawer contains 100 red socks, 80 green socks, 60 blue socks, and 40 black socks. Bo selects socks one at a time from the drawer, but is unable to see the colors of the socks. What is the smallest number of socks that must be selected to guarantee that the selection contains at least 10 pairs?

- (a) 20 (b) 21 (c) 22 (d) 23 (e) 80 [3] _____
-

4. It's true that $\ln(x + 3) \geq 0$ if

- (a) $-3 < x < -2$ (b) $x \geq 1$ (c) $x \geq -2$ (d) $x > 0$ (e) $x \geq 4$ [4] _____
-

5. Given that $\cos x = \frac{1}{2}$ and that $-90^\circ < x < 90^\circ$, which one of the following has the largest value?

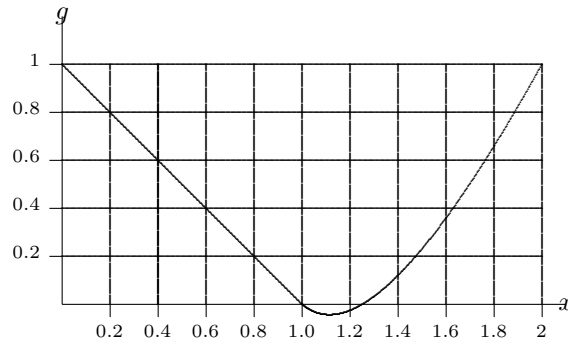
- (a) $\sin x$
(b) $\cos x$
(c) -2
(d) Answers (a), (b) & (c) all equal.
(e) Impossible to tell. [5] _____
-

6. If the amplitude of $y = \left(\frac{1}{k}\right) \cos(k^2\theta)$ is 2, then the period must be

- (a) π (b) 2π (c) 4π (d) 8π (e) 16π [6] _____
-

Go to back \Rightarrow

7. The graph below is that of $y = g(x)$. Compute $g(g(2))$.



- (a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (e) 0 [7] _____
-

8. Which one of the following statements is **true** about the solutions of $4x^2 + 6x + 9 = 9 - 2x$?

- (a) The sum of the solutions is less than their product.
(b) The sum of the solutions is equal to their product.
(c) The sum of the solutions is greater than their product.
(d) The product of the solutions is a complex number.
(e) None of these. [8] _____
-

9. Notice that the vowels in the word FACETIOUS are in alphabetic order. Consider all the possible new words made from rearrangements of the letters in this word. In how many of them are the vowels in alphabetic order?

Note: $C(n, r) = \frac{n!}{r!(n-r)!}$

- (a) $C(9, 5)$ (b) $C(9, 5) \cdot 4!$ (c) $C(9, 4)$ (d) $C(9, 4) \cdot 5!$ (e) $4! \cdot 5!$ [9] _____
-

10. The integral $\int_a^b x^2 e^{x^3} dx$ is equivalent to which one of the following integrals?

- (a) $\frac{1}{3} \int_{a^3}^{b^3} e^u du$ (b) $\int_{a^3}^{b^3} e^u du$ (c) $\frac{1}{3} \int_a^b e^u du$ [10] _____
(d) $-\frac{1}{3} \int_a^b e^u du$ (e) $3 \int_{a^3}^{b^3} e^u du$
-

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Test 4

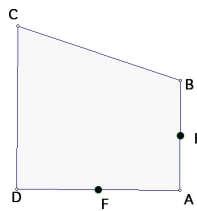
NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. In the figure below, E is the midpoint of \overline{AB} and F is the midpoint of \overline{AD} . If the area of the quadrilateral $FAEC$ is 13 square inches, what is the area of $ABCD$?



- (a) 19.5 in^2 (b) 26 in^2 (c) $26\sqrt{2} \text{ in}^2$ (d) 39 in^2 (e) 52 in^2 [1] _____

2. If $9e^{3t} = 27$, then $t =$

- (a) $\frac{\ln 27}{27}$ (b) $\ln \sqrt[3]{3}$ (c) 1 (d) $\ln 3$ (e) $\ln 9$ [2] _____

3. $\frac{d}{dx} (\cos(9\pi)^{30}) =$

- (a) $270 (\cos(9\pi))^{30} \cdot \sin(9\pi)$ (b) $30 (\cos(9\pi))^{30}$ (d) 0
 (c) $30 (\cos(9\pi))^{30} \cdot \sin(9\pi)$ (e) the function is not differentiable. [3] _____

4. The Braves and the Yankees are playing in the World Series. The World Series is set of games that continue until one team has won four games. Each team has a probability of $\frac{1}{2}$ of winning each game. Given that exactly six games are played, what is the probability that the Braves win the World Series?

- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{7}{16}$ (d) $\frac{9}{32}$ (e) $\frac{15}{64}$ [4] _____

5. A garden is fenced in the shape of a regular hexagon with each side 10 meters in length. A dog is tied with a 12-meter leash to a post at one of the vertices of the hexagon, on the outside of the garden. What area outside the garden can the dog cover?

- (a) 72π (b) 96π (c) $66\frac{2}{3}\pi$ (d) $97\frac{1}{3}\pi$ (e) 144π [5] _____

Go to back \Rightarrow

6. Which graphing window will show exactly one period of the curve $y = 3 \sin(2x) + 4$?

- (a) $[0, 2\pi] \times [3, 4]$ (b) $[0, \pi] \times [1, 7]$ (c) $[0, 2\pi] \times [-2, 3]$

[6] _____

- (d) $[0, \pi] \times [-3, 4]$ (e) None of the above.
-

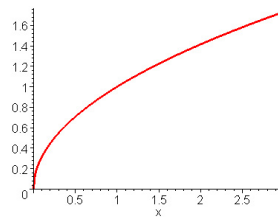
7. An encoding system is being designed for the English alphabet. The first few letters are encoded as shown below. Following the same pattern, what would be the code for the letter Z?

Letter	A	B	C	D	E	F	G	H	I
Code Word	aaaaa	aaaab	aaaba	aaabb	aabaa	aabab	aabba	aabbb	abaaa

- (a) bbbbbb (b) bbbba (c) bbaab (d) bbaaa (e) abbbb

[7] _____

8. According to the graph of $p(x)$ below, $p^{-1}(1.4)$ is approximately



- (a) $\frac{1}{1.4}$ (b) 1.3 (c) 2 (d) 3 (e) $\frac{1.4}{1}$

[8] _____

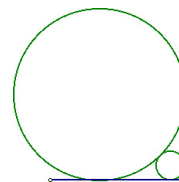
9. Given $f(x) = \int_{3x}^{x^2} \sin(t^2) dt$, find $f'(x)$.

- (a) $2x \cos(x^4) - 3 \sin(9x^2)$ (b) $\cos(x^4)$ (c) $\sin(x^2)$

- (d) $2x \sin(x^4) - 3 \sin(9x^2)$ (e) $2x \cos(x^4)$

[9] _____

10. In the figure below, the two circles are tangent to the right angle and to one another. If the larger circle has radius 1, what is the radius of the smaller circle?



- (a) $1 + \sqrt{2}$ (b) $\sqrt{2} - 1$ (c) $2\sqrt{2} - 1$ (d) $3 + 2\sqrt{2}$ (e) $3 - 2\sqrt{2}$

[10] _____

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Test 5

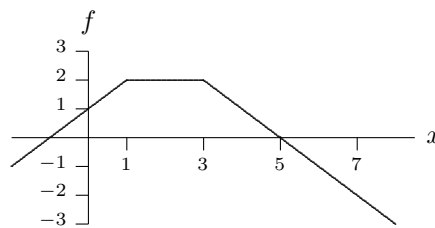
NAME: _____

CLASS 11 & 12 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The function $f(x)$ is defined by the picture below. Evaluate the integral $\int_1^7 f(x) dx$.



- (a) 0 (b) 4 (c) 5 (d) 6 (e) 7 [1] _____
-

2. Determine the positive integer value of $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}}$.

- (a) 2 (b) 3 (c) 4 (d) 5 (e) 6 [2] _____
-

3. When $10^{93} - 93$ is expressed as a single whole number, what is the sum of the digits?

- (a) 10 (b) 93 (c) 819 (d) 826 (e) 833 [3] _____
-

4. If $f(t)$ is measured in meters per second and t is measured in seconds, what are the units of its derivative $f'(t)$?

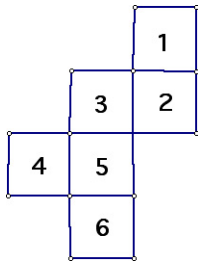
- (a) no units (b) meters (c) meters per second (d) meters per second² (e) seconds [4] _____
-

5. The graphs of $y = f(x)$ and its inverse $y = f^{-1}(x)$ are symmetric with respect to which line?

- (a) $y = 0$ (b) $x = 0$ (c) $y = x$ (d) $y = -x$ (e) $y = \frac{1}{x}$ [5] _____
-

Go to back⇒

-
6. When the net of squares below is cut out and folded to form a cube, what is the product of the numbers on the four faces adjacent to the one labeled with a 1?

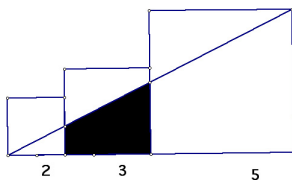


- (a) 120 (b) 144 (c) 180 (d) 240 (e) 360 [6] _____
-

7. Which statement is the **best** description of location of the vertex of the quadratic function $f(x) = ax^2 + bx + c$, with $a > 0$ and $b^2 - 4ac > 0$?

- (a) It will lie on the x -axis.
(b) It will lie on the y -axis.
(c) It will lie below the x -axis, on the y -axis.
(d) It will lie above the x -axis, on the y -axis.
(e) It will not lie on an axis. [7] _____
-

8. Three squares have the dimensions (in inches) indicated in the diagram below. What is the area of the shaded quadrilateral?



- (a) 6.00 in^2 (b) 5.25 in^2 (c) 4.25 in^2 (d) 4 in^2 (e) 3.75 in^2 [8] _____
-

9. If $-1 < m < 0$, then $\arcsin(m)$ is

- (a) negative (b) zero (c) positive (d) is undefined (e) none of these [9] _____
-

10. A merchant pays duty on certain good at three different places. At the first place he gives one third of the goods, at the second place he gives one fourth of the remainder, and at the third place, he gives one fifth of the remainder. The total duty is 24. What was the original amount of goods?

- (a) 32 (b) 36 (c) 40 (d) 42 (e) 44 [10] _____
-

Grades 11-12
2006 Math Contest Exam

Exam	T1	T2	T3	T4	T5
P1	a	c	c	b	b
P2	b	e	a	b	b
P3	d	b	d	d	d
P4	c	b	c	a	d
P5	e	a	e	d	c
P6	a	d	d	b	b
P7	e	b	e	c	e
P8	c	d	a	c	b
P9	a	e	b	d	a
P10	e	d	a	e	c