

WCTM MATHEMATICS CONTEST, 2005

Test 1

NAME: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. What is the 532nd term of the sequence G,R,E,A,T,G,R,E,A,T,G,R,E,...?

- (a) G (b) R (c) E (d) A (e) T [1] _____

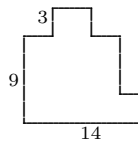
2. Sue's change jar contains quarters, nickels, and dimes. There are 47 coins whose total worth is \$7.10. The number of quarters in the jar is 4 less than twice the number of nickels. How many dimes does Sue have?

- (a) 22 (b) 15 (c) 12 (d) 7 (e) 4 [2] _____

3. Bob's salary is \$53,820 after a 15% raise and then a 10% decrease in pay. What was Bob's (approximate) salary before the raise?

- (A) \$50,322 (B) \$51,129 (C) \$52,000 (D) \$56,511 (E) \$56,626 [3] _____

4. The adjacent sides of the decagon shown below meet at right angles. What is its perimeter?

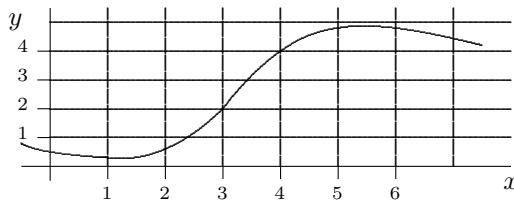


- (a) 26 (b) 32 (c) 48 (d) 52 (e) 56 [4] _____

5. The graph shown here defines f over the interval $[0,6]$. Arrange the following values in increasing order.

$0, f'(2), f'(3), f'(5), f''(5)$

Which value is in the middle of your list?



- (a) 0 (b) $f'(2)$ (c) $f'(3)$ (d) $f'(5)$ (e) $f''(5)$ [5] _____

Go to back \implies

6. The supplement of an angle is 2.5 times the complement of the angle. The sum of the supplement and complement angles is:

- (a) 120 (b) 150 (c) 180 (d) 210 (e) 270 [6] _____
-

7. The difference between two numbers is 36. If the larger number is divided by the smaller number the quotient is 5 and the remainder is 4. What is the sum of the two numbers?

- (a) 42 (b) 44 (c) 52 (d) 70 (e) 100 [7] _____
-

8. Find a polynomial, with integer coefficients, that has zeros at $x = \frac{1}{2}$, $x = 0$ and $x = 5i$.

- (a) $x^2 - 5x$ (b) $2x^2 - x$ (c) $2x^3 - (1 + 10i)x^2 + 5ix$
(d) $23x^3 - 11x^2 + 5x$ (e) $2x^4 - x^3 + 50x^2 - 25x$ [8] _____
-

9. Which equation represents the line that has a slope of $\frac{3}{2}$ and passes through the point of intersection of the graphs of $x - y = 5$ and $3x + 10y = 2$?

- (a) $y = \frac{3}{2}x - 7$ (b) $y = \frac{3}{2}x - 5$ (c) $y = \frac{3}{2}x - 1$ (d) $y = \frac{3}{2}x$ (e) $y = \frac{3}{2}x + 4$ [9] _____
-

10. Determine the median of the set numbers $\{8, 6.5, 8.5, 10, 8, 2.5, 2\}$.

- (a) 6.5 (b) 7.5 (c) 8 (d) 8.5 (e) 10 [10] _____
-

Test 2

NAME: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

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1. The denominator of a fraction is 2 more than 3 times the numerator. If 1 is added to the denominator, the resulting fraction is equivalent to $\frac{1}{4}$. Find the original fraction.

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{4}{15}$ (d) $\frac{3}{11}$ (e) $\frac{3}{6}$ [1] _____

2. The Crazy Store increases all of its prices by 25%. The next day the store decreases all prices by 20%. What is the net percent change in price?

(a) 0% (b) 4% (c) 5% (d) 10% (e) 45% [2] _____

3. How many terms are in the sequence 7,10,13,16,..., 2962?

(a) 985 (b) 986 (c) 987 (d) 2955 (e) 2962 [3] _____

4. In Bayview High School 105 students received 61 district and state math awards. The breakdown by gender is shown below. Find the probability that a student is a girl given that the student won a state math award.

	District	State
Boys	52	29
Girls	53	32

(a) $\frac{32}{53}$ (b) $\frac{32}{61}$ (c) $\frac{53}{61}$ (d) $\frac{32}{105}$ (e) $\frac{61}{105}$ [4] _____

5. If m and p are positive integers and $(m + p) \cdot m$ is even, which one of the following must be true?

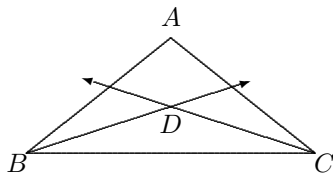
(a) If m is odd, then p is odd
(b) If m is odd, then p is even
(c) If m is even, then p is even
(d) If m is even, then p is odd
(e) m must be even [5] _____

6. The set S consists of all numbers that are the sum of three consecutive prime numbers. The smallest prime number in S is

(a) 3 (b) 6 (c) 10 (d) 23 (e) 31 [6] _____

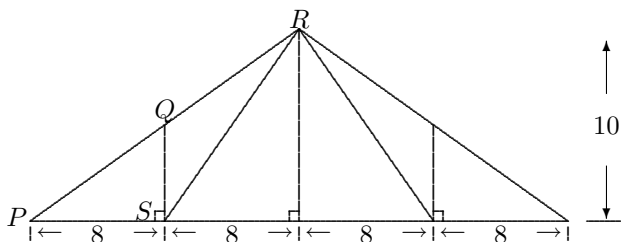
Go to back \implies

7. In the triangle ABC , the bisectors of $\angle ABC$ and $\angle ACB$ meet at point D . The degree measure of $\angle BDC$ is 100° . What is the degree measure of $\angle BAC$?



- (a) 10° (b) 20° (c) 30° (d) 80° (e) 100° [7] _____

8. Q is the point on the line segment \overline{PR} in the truss shown below. What is the sum of the lengths of \overline{PQ} and \overline{QS} ?



- (a) $18\sqrt{3}$ (b) $\sqrt{97}$ (c) $16 + 3\sqrt{2}$ (d) $5 + \sqrt{89}$ (e) $9 + \sqrt{89}$ [8] _____

9. How many diagonals does a regular dodecagon contain?

- (a) 12 (b) 54 (c) 78 (d) 108 (e) 156 [9] _____

10. From the table of values for f, g, f' and g' , find $H'(1)$ if $H(x) = g(f(x))$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

- (a) 4 (b) 6 (c) 9 (d) 24 (e) 36 [10] _____

Test 3

NAME: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Two sides of a non-isosceles triangle measure 8 cm. and 12 cm. If the length, in centimeters, of the third side is also an integer, how many choices are there? Exclude triangles with zero area.

(a) 13 (b) 15 (c) 18 (d) 19 (e) infinitely many [1] _____

2. A 19-liter mixture consists by volume of 1 part juice to 18 parts water. If x liters of juice and y liters of water are added to this mixture to make a 54-liter mixture consisting of 1 part juice to 2 parts water, what is the value of x ?

(a) 17 (b) 18 (c) 27 (d) 35 (e) 36 [2] _____

3. A circular target with a radius of 12 inches is to have a circular bull's-eye painted in the center so that the area of the bull's-eye is 2% of the total area of the target. The radius of the bull's-eye in inches should be:

(a) 0.24 (b) 1.2 (c) $\sqrt{2.88}$ (d) $\frac{1}{\sqrt{\pi}}$ (e) $\frac{12}{\sqrt{145}}$ [3] _____

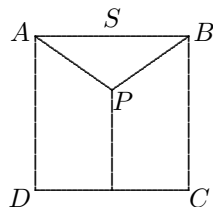
4. What is the next term in the following sequence: 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, ...?

(a) 1705 (b) 1431 (c) 1444 (d) 2032 (e) 1350 [4] _____

5. A team of two men and four women is formed at random from a larger group of nine men and five women. Damon is one of the men and Latisha is one of the women. How many different teams include both Damon and Latisha?

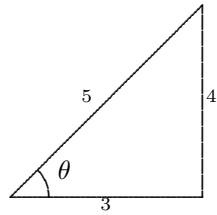
(a) 8640 (b) 192 (c) 180 (d) 45 (e) 32 [5] _____

6. $ABCD$ is a square with $\overline{AB} = S$. Point P is an interior point of the square such that \overline{AP} , \overline{BP} , and the distance from point P to \overline{CD} are all equal. How long is \overline{AP} ?



(a) $\frac{1}{2}S$ (b) $\frac{3}{5}S$ (c) $\frac{5}{8}S$ (d) $\frac{2}{3}S$ (e) $\frac{3}{4}S$ [6] _____

7. Compute $\cos 2\theta$.



- (a) $-\frac{7}{25}$ (b) $\frac{3}{5}$ (c) $\frac{8}{5}$ (d) $\frac{7}{25}$ (e) $\frac{8}{25}$ [7] _____
-

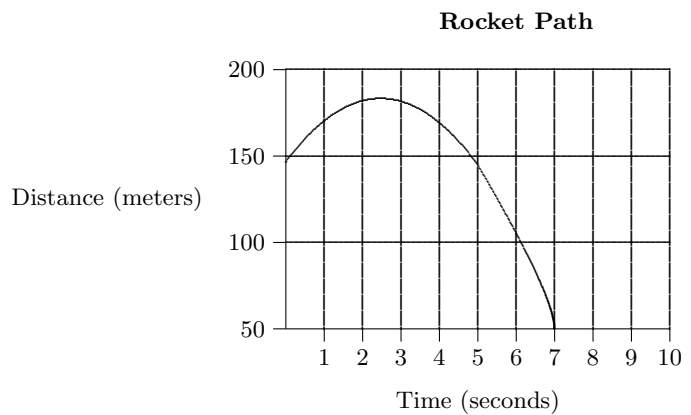
8. Select a description for a pair of values for a and b for which $a > 0$, $b > 0$, and $\log_a b = \log_b a$.

- (a) $a = 1$ and $b = 10$ (b) $a = e$ and $b = 1$ (c) $a = \frac{1}{b}$ [8] _____
(d) $a = -b$ (e) $a = e$ and $b = 10$
-

9. If $g(x) = x^3$, then $\lim_{x \rightarrow 2} \frac{g(x+2) - g(2)}{x - 2} =$

- (a) -2 (b) 0 (c) 1 (d) 12 (e) ∞ [9] _____
-

10. The graph defines the height of a rocket. Select the most reasonable average velocity of the rocket between 1 and 4 seconds from the choices below.



- (a) -80 m/s (b) -120 m/s (c) 0 m/s (d) 80 m/s (e) 120 m/s [10] _____
-

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. If $f(-10) = 12$, $g(1) = 14$, and $h(x) = 3f(4 - g(x - 2))$, find $h(3)$.

- (a) 12 (b) 14 (c) 36 (d) 42 (e) -30 [1] _____

2. For what integer n does $\frac{1}{6^n + 6^n + 6^n + 6^n + 6^n + 6^n} = 6^{36}$

- (a) -37 (b) -36 (c) -10 (d) -6 (e) 0 [2] _____

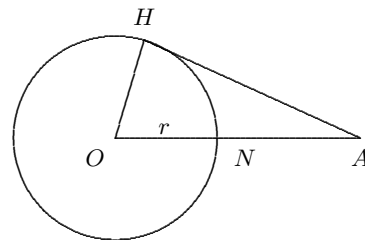
3. Solve for x : $A = \left(3 + \frac{1}{x}\right)^{2x}$

- (a) $\frac{\ln A}{2 \ln \left(3 + \frac{1}{A}\right)}$ (b) $\frac{1}{\sqrt[2x]{A} - 3}$ (c) $\frac{\ln A}{2 \ln \left(3 + \frac{1}{x}\right)}$ (d) A (e) can't be solved [3] _____

4. $S = \frac{a}{b} + \frac{c}{d} + \frac{1}{e}$ and $0 < a < b < c < d < e$. Adding 1 to which variable results in the greatest increase in S ?

- (a) a (b) b (c) c (d) d (e) e [4] _____

5. In the figure O is the center of the circle, \overline{AH} is the tangent to circle at H , and $ON = NA$. Find, in terms of r , the area of the region inside $\triangle OAH$ and outside the circle.



- (a) $\frac{(3\sqrt{5} - \pi) r^2}{6}$ (b) $\frac{(3\sqrt{2} - \pi) r^2}{6}$ (c) $\frac{(\sqrt{5} - \pi) r^2}{6}$
 (d) $\frac{(2\sqrt{3} - \pi) r^2}{6}$ (e) $\frac{(3\sqrt{3} - \pi) r^2}{6}$

[5] _____

6. Determine the period of the function $f(x) = -4 \cos\left(\pi x + \frac{2\pi}{3}\right)$.

- (a) $-\frac{2\pi}{3}$ (b) 2π (c) π (d) 4 (e) 2 [6] _____

7. An object is moving along a horizontal number line such that its position at time t is given by:

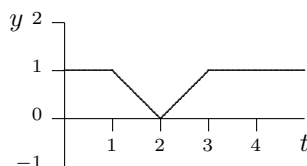
$$p(t) = 25 \sin\left(\frac{\pi t}{5}\right).$$

When $t = 4$, which of the following is the best statement about the particle?

- (a) It is moving to the right and speeding up
- (b) It is moving to the right and slowing down
- (c) It is moving to the left and speeding up
- (d) It is moving to the left and slowing down
- (e) It is not moving

[7] _____

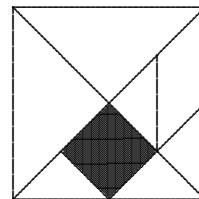
8. Let $g(x) = \int_0^x f(t) dt$ and let $f(t)$ be defined the curve shown below. Find $g(3)$.



- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 4

[8] _____

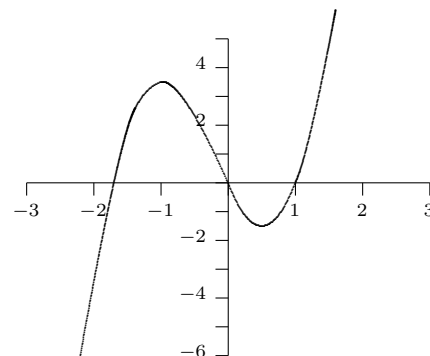
9. What is the ratio of the area of the shaded square to the area of the large square?



- (a) $\frac{1}{6}$
- (b) $\frac{1}{7}$
- (c) $\frac{1}{8}$
- (d) $\frac{1}{12}$
- (e) $\frac{1}{16}$

[9] _____

10. The graph of $f'(x)$ is shown on the right. Where is the graph of $f(x)$ concave up?



- (a) $(-\infty, \infty)$
- (b) $(-\infty, -1.7) \cup (0, 1)$
- (c) $(-1.7, 0) \cup (1, \infty)$
- (d) $(-\infty, -1) \cup (0.5, \infty)$
- (e) $(-1, 0.5)$

[10] _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. If $f(x) = 3^{kx}$ and $f(2) = 10$, then:

- (a) $f(4) = 100$ (b) $f(3) + f(1) = 30$ (c) $f(1) = 5$
 (d) $f(3) - f(1) = f(2)$ (e) $k = \ln \sqrt{10}$

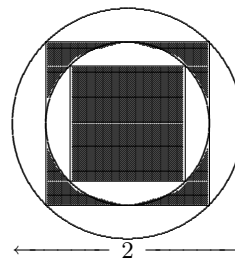
[1] _____

2. Find the following limit if it exists. $\lim_{x \rightarrow n} \frac{2x^2 - 5xn + 3n^2}{x - n}$

- (a) n (b) $-n$ (c) 0 (d) $2n$ (e) does not exist

[2] _____

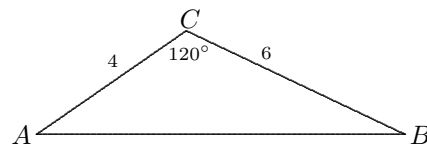
3. A point is selected at random from the figure made from two concentric circles and two concentric squares shown here. The outer circle has $r = 1$. Find the probability of that point falling in the shaded area.



- (a) $\frac{6 - \pi}{2\pi}$ (b) 1 (c) $\frac{2\pi - 2}{\sqrt{\pi}}$ (d) $\frac{\pi + 3}{2\pi}$ (e) $\frac{\pi - 1}{\sqrt{\pi}}$

[3] _____

4. Find the area of the triangle ABC with $\angle C = 120^\circ$, $\overline{AC} = 4$, and $\overline{BC} = 6$.



- (a) $3\sqrt{3}$ (b) $6\sqrt{2}$ (c) $6\sqrt{3}$ (d) 12 (e) $12\sqrt{3}$

[4] _____

5. Evaluate $\cos\left(\arcsin \frac{x}{7}\right)$ where $0 < x < 7$.

- (a) $\frac{\sqrt{49 - x^2}}{7}$ (b) $\frac{7}{x}$ (c) $\frac{x\sqrt{7}}{49}$ (d) $\frac{7}{\sqrt{x}}$ (e) $\frac{\sqrt{x^2 - 49}}{7}$

[5] _____

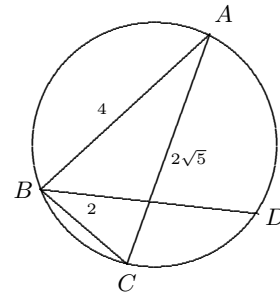
6. If we call 5 the first term, then what is the twenty-seventh term of the sequence 5, 8, 7, 11, 9, 14, ...?

- (a) 29 (b) 31 (c) 33 (d) 44 (e) 47 [6] _____
-

7. A rational function has two zeros: one is at $x = 3$ and the other is at $x = -2$. It also has a horizontal asymptote at $y = 5$. Which statement best describes how many vertical asymptotes this rational function might have?

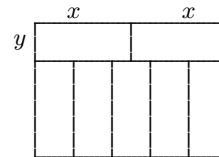
- (a) Only 0 (b) Only 1 (c) 1 or 2 (d) Only 2 (e) 0, 1 or 2 [7] _____
-

8. Points A, B, C , and D lie on a circle with $AB = 4$ and $BC = 2$; \overline{AC} is a diameter; and $\angle ABD \equiv \angle CBD$. What is the length of \overline{BD} ?



- (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $2\sqrt{5}$ (d) $3\sqrt{2}$ (e) $\sqrt{10}$ [8] _____
-

9. A rectangle with perimeter 272 is divided into seven congruent rectangles as shown in the diagram. What is the perimeter of one of the seven congruent rectangles?



- (a) $\frac{272}{7}$ (b) 112 (c) $\frac{1116}{7}$ (d) 224 (e) 784 [9] _____
-

10. How many distinct solutions does the equation $\det(A - xI) = 0$ have when I is the 4 x 4 identity matrix, \det is the determinant and

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 3 & 0 \\ 1 & 0 & 0 & 2 \end{pmatrix}$$

- (a) none (b) one (c) two (d) three (e) four [10] _____
-

Answer Key – AB Test 2005

	1	2	3	4	5	6	7	8	9	10
test 1	b	b	c	d	d	d	c	e	a	c
test 2	d	a	b	b	a	d	b	d	b	e
test 3	a	a	c	a	e	c	a	c	d	c
test 4	c	a	e	a	e	e	d	c	c	d
test 5	a	b	a	c	a	b	e	c	b	d