Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007

Test 1 **NAME**: _____

CLASS 7 & 8 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. I am a two-digit number. When I am multiplied by the sum of my digits, the result is 70. In which interval can I be found?

   (A) $[10, 20]$ (B) $[21, 30]$ (C) $[31, 40]$ (D) $[41, 50]$ (E) $[51, 60]$ [1] _____

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2. What is the tenth number in the following sequence?

$$\frac{1}{5},\ \frac{4}{7},\ 1,\ 1\frac{5}{11},\ 1\frac{12}{13},\ \ldots$$

   (A) $1\frac{3}{23}$ (B) $1\frac{22}{23}$ (C) $2\frac{6}{23}$ (D) $3\frac{11}{23}$ (E) $4\frac{8}{23}$ [2] _____

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3. In how many different arrangements can five keys be placed on a circular key ring? No two keys are alike.

   (A) 12 (B) 20 (C) 24 (D) 60 (E) 120 [3] _____

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4. When the hour hand on a clock is between 12:00 and 1:00, during which 1-minute interval are the minute hand and the hour hand pointed in exactly opposite directions?

   (A) 12:30−12:31 (B) 12:31−12:32 (C) 12:32−12:33 (D) 12:33−12:34 (E) 12:34−12:35 [4] _____

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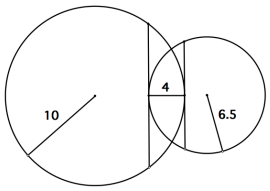
5. Circles with 20 cm and 13 cm diameters overlap 4 cm as shown below in the figure. Find the sum of the lengths of the tangent chords.



   (A) 8.5 (B) 14 (C) 20.5 (D) $16 + 6.5\sqrt{3}$ (E) 28 [5] _____

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6. The digits 1, 3, 4, 6, 8, and 9 are each used exactly once to form three two-digit primes. What is the sum of the three primes?

(A) 148 (B) 166 (C) 175 (D) 193 (E) 220 [6] _____

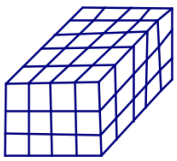
7. Create six different three-digit numbers from the digits 1, 2, and 3. What is their mean?

(A) 219 (B) 222 (C) 227 (D) 232.3 (E) 340.3 [7] _____

8. The Millers' house is valued at \$80,000. Suppose that the value increases by 5% per year. How much will it be worth in three years? Round to the nearest dollar.

(A) \$84,000 (B) \$88,200 (C) \$92,610 (D) \$97,241 (E) \$252,000 [8] _____

9. A stack of unit cubes forms a rectangular prism 3 units by 4 units by 5 units. If the faces on the outside of the stack are painted, how many of the unit cubes in the stack will have no paint on any of their faces?



(A) 4 (B) 6 (C) 10 (D) 12 (E) 30 [9] _____

10. A circular pizza with a 14 inch diameter has a 1 inch border of crust. There are no toppings on the crust. What percentage of the pizza has no toppings?

(A) 7.1% (B) 13.8% (C) 14.3% (D) 14.8% (E) 26.5% [10] _____

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Test 2 **NAME**: _____

CLASS 7 & 8 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. Let $a_n = n^3 + 3n^2 + 2n$. For what value of n does $a_n = 456,456$?

   (A) 56 (B) 76 (C) 456 (D) 1001 (E) 76076 [1] _____

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2. At the conclusion of a committee meeting, a total of 28 handshakes were exchanged. Assuming that each person was equally polite toward all the others, how many people were present?

   (A) 7 (B) 8 (C) 14 (D) 16 (E) 27 [2] _____

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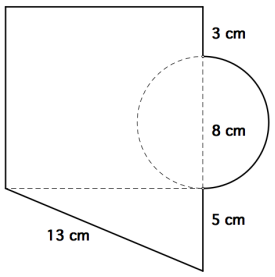
3. Find the area of the region below. It is made from a triangle, a rectangle, and a circle. The interior boundaries of each are dashed to help you visualize the parts.



   (A) $(162 + 8\pi)$ cm (B) $(114 + 16\pi)$ cm (C) $(84 + 8\pi)$ cm

   (D) $(162 + 16\pi)$ cm (E) $(173 + 16\pi)$ cm [3] _____

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4. 10 percent of the Americans surveyed prefer dogs to cats, and four out of five people surveyed who prefer dogs are men. What percent of women prefer dogs?

   (A) 10% (B) 4% (C) 2% (D) 0.5% (E) 0.1% [4] _____

Go to back⟹

5. Sam can buy three oranges or two apples for the same amount of money. How many apples can Sam buy if she has just enough for 24 oranges?

(A) 8 (B) 12 (C) 16 (D) 18 (E) 20 [5] _____

6. The line through the points $(m, -9)$ and $(7, m)$ has slope m. What is the value of m?

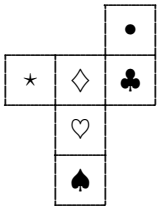
(A) 3 (B) -3 (C) 0 (D) 4 (E) there is no value [6] _____

7. In the following addition problem, each letter represents a different digit. What is the value of $A + B + C$?

$$\begin{array}{r} ABC \\ ABC \\ +ABC \\ \hline BBB \end{array}$$

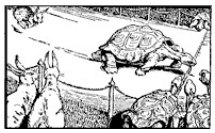
(A) 8 (B) 10 (C) 12 (D) 13 (E) 32 [7] _____

8. If the figure shown were folded to form a cube, what symbol would be across from the $\bullet$?



(A) $\diamondsuit$ (B) $\spadesuit$ (C) $\star$ (D) $\heartsuit$ (E) $\clubsuit$ [8] _____

9. A tortoise and a hare start at the same time in a 10-mile race. The hare runs for 20 minutes at 12 mph; stops to rest; falls asleep for 70 minutes; awakes; and finishes the race, again running at 12 mph. Meanwhile, the tortoise plugs along at the same constant rate throughout the race. If they reach the finish line at the same time, at what constant rate does the tortoise run?



(A) 0.5 mph (B) 2 mph (C) 3.5 mph (D) 5 mph (E) 6 mph [9] _____

10. The third term in an arithmetic sequence is 15, and the fifth term is 23. What is the first term?

(A) -1 (B) 3 (C) 7 (D) 9 (E) 11 [10] _____

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007

Test 3 **NAME**: _____

CLASS 7 & 8 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. Consider the sum shown below. What is its hundreds place digit?

$$7 + 77 + 777 + 7777 + \ldots + 77777777777777777$$

(A) 0 (B) 1 (C) 4 (D) 7 (E) 9 [1] _____

---

2. What fractional part of the figure below is shaded?

(A) $\frac{1}{4}$ (B) $\frac{5}{8}$ (C) $\frac{5}{13}$ (D) $\frac{5}{18}$ (E) $\frac{9}{20}$ [2] _____

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3. On a visit to a kite factory, Miguel noticed that the kites are made in the following order, and then repeated: dragon, box, bat, fish, tiger, turkey. What design will be on the eighty-eighth kite?

(A) fish (B) tiger (C) turkey (D) dragon (E) bat [3] _____

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4. What is the remainder when 1999 raised to the 200th power is divided by 5?

(A) -1 (B) 0 (C) 1 (D) 5 (E) 9 [4] _____

---

5. A bag contains five red marbles, three green marbles, and four blue marbles. How many green marbles must be added so that the probability of drawing a green marble at random is $\frac{3}{4}$?

(A) 9 (B) 12 (C) 15 (D) 16 (E) 24 [5] _____

Go to back⟹

6. Five seventh-grade friends are standing in line to pick up their class schedules.

   - Laura is the 20th person behind Rayma.
   - Rayma is the 50th person ahead of Deanna.
   - Deanna is the 10th person behind Charlotte.
   - Shelly is the 30th person ahead of Laura.
   - Charlotte is the 50th person behind Shelly.

   If there are 50 students in front of the friend who is closest to the front, and 50 people are behind the friend closest to the back of the line, how many people are in line?

   (A) 155 (B) 156 (C) 160 (D) 161 (E) 170 [6] _____

---

7. Keith has an equal number of nickels, dimes, and quarters. The total value of his coins is \$8.00. How many of each coin does he have?

   (A) 10 (B) 20 (C) 30 (D) 40 (E) 60 [7] _____

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8. On a test, a student answered 15 of the first 20 questions correctly. Of the remaining questions, he answered exactly one-third correctly. If all the questions were worth the same number of points and the student's grade on the test was 50 percent, how many questions were on the test?

   (A) 20 (B) 30 (C) 40 (D) 50 (E) 60 [8] _____

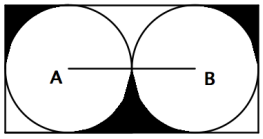
---

9. The term $\frac{a}{b}$ is a fraction. If 2 is added to the numerator, the value of the fraction is $\frac{1}{2}$. If 3 is added to the denominator, the fraction has a value of $\frac{1}{3}$. Find the value of the sum $a+b$.

   (A) 5 (B) 7 (C) 18 (D) 21 (E) 25 [9] _____

---

10. A rectangle is drawn around two congruent circles as shown below. Points A and B are the centers of these two circles. If $\overline{AB} = 10$, find the approximate total area of the shaded regions.



   (A) 5.4 (B) 10.8 (C) 21.5 (D) 43 (E) 78.5 [10] _____

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Test 4 **NAME**: _____

CLASS 7 & 8 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. Joan has the same number of sisters as she has brothers, but her brother Matt has twice as many sisters as he has brothers. If their family has fewer than 10 children, how many children are there?

   (A) 5 (B) 6 (C) 7 (D) 8 (E) 9 [1] _____

---

2. On a line:

   - Point B is 1 unit to the right of point A.
   - Point C is 2 units to the left of point A.
   - Point D is 3 units to the right of point B.
   - Point E is 12 units to the left of point D.

   How many units is the distance from A to E?

   (A) 2 (B) 6 (C) 8 (D) 12 (E) 18 [2] _____

---

3. Five paper bags contain a total of thirty apples.

   - The first and second bags contain a total of fourteen apples.
   - The second and third contain a total of ten apples.
   - The third and fourth contain a total of nine apples.
   - The fourth and the fifth contain a total of twelve apples.

   How many apples are in the fifth bag?

   (A) 4 (B) 5 (C) 6 (D) 7 (E) 8 [3] _____

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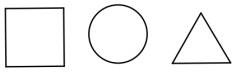
4. Two traffic lights turn red together at exactly 5:00 PM. One light is on a 36-second cycle from red back to red, and the other is on a 48-second cycle. At what time will they again turn red together? Give the answer exact to seconds.

   (A) 5:01:12 (B) 5:01:24 (C) 5:02:08 (D) 5:02:12 (E) 5:02:24 [4] _____

Go to back⟹

5. In a kindergarten class, each child is given a red crayon, a green crayon, a blue crayon, and the following picture:



Each child is then asked to color each of the shapes in the picture. They are told to color each figure with only one color, and no two figures have the same color. In how many different ways can the children color the picture?

(A) 3 (B) 4 (C) 6 (D) 9 (E) 12 [5] _____

---

6. The following numbers are the first five terms in a pattern: 2, 6, 12, 20, 30. What is the tenth term?

(A) 34 (B) 38 (C) 42 (D) 90 (E) 110 [6] _____

---

7. If Jane has only dimes and nickels, in how many different ways can she make change for a dollar? Assume she always has enough coins to make the change.

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12 [7] _____

---

8. There is a least possible natural number value for k for which $k = \frac{n}{3} + \frac{n}{4}$, where n is a whole number. Multiply k by seven, and then sum the digits of the result.

(A) 3 (B) 10 (C) 12 (D) 13 (E) 17 [8] _____

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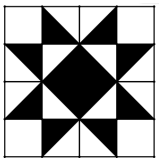
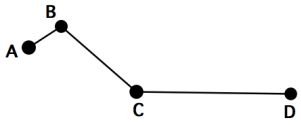
9. Consider the eight-point-star quilt block shown below. How many lines of symmetry does the block have?



(A) 0 (B) 1 (C) 2 (D) 4 (E) 8 [9] _____

---

10. The distance from C to D is 24 miles. The distance from B to C is $\frac{2}{3}$ of the distance from C to D. The distance from A to B is $\frac{3}{8}$ of the distance from B to C. What is the distance from A to B?



(A) 4 (B) 6 (C) 9 (D) 16 (E) 46 [10] _____

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Test 5 **NAME**: _____

CLASS 7 & 8 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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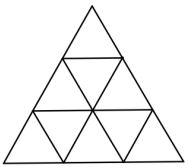
1. We will define an isosceles trapezoid as a quadrilateral, or four-sided polygon, with exactly one set of parallel-sides and a pair of congruent base angles. How many different isosceles trapezoids can you count in the figure below?



(A) 3 (B) 12 (C) 15 (D) 18 (E) 21 [1] _____

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2. Bekah has three brass house number digits: 2, 3, and 5. How many distinct numbers can she form using one or more of the digits?

(A) 6 (B) 7 (C) 9 (D) 12 (E) 15 [2] _____

---

3. Find the value of the expression below.

$$\frac{1}{4+\frac{1}{4+\frac{1}{4\frac{1}{4}}}}$$

(A) $\frac{1}{4}$ (B) $\frac{4}{17}$ (C) $\frac{17}{72}$ (D) $\frac{72}{288}$ (E) $\frac{72}{305}$ [3] _____

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4. A player's season-home-run total is assumed to be proportional to any mid-season total. After 108 games in 1999, Mark McGwire had accumulated 42 home runs. At that point, what was his projected season-home-run total? There were 162 games are in the 1999 season.

(A) 23 (B) 63 (C) 75 (D) 87 (E) 91 [4] _____

---

5. Determine a value of k so that $2x+1$ is a factor of $2x^2+7x+k$.

(A) -4 (B) -3 (C) 1 (D) 2 (E) 3 [5] _____

Go to back⟹

6. A speedboat costs $35,000. The salesman says that the boat will *depreciate* in value by 10% each year. In other words, each year the boat will be worth 10% less than the year before. How many years must pass before the boat is worth only half of the original price?

(A) 4.6 (B) 5 (C) 6.6 (D) 7.2 (E) 8.3 [6] _____

---

7. The pups dug into dinner. Polaris ate more than Sirius. Castor ate less than Pollux. Pollux ate less than Sirius, but more than Rigel. Which pup ate the second most?

(A) Castor (B) Polaris (C) Pollux (D) Rigel (E) Sirius [7] _____

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8. The Ice Cream Shop offers five different flavors daily. Today's flavors include vanilla, chocolate, strawberry, cookies and cream, and raspberry. Homemade waffle, pretzel, and sugar cones are also available. If Jane orders a double-dip cone, from how many different two flavor ice cream cones can she choose? Make your count assuming that the order of the dips doesn't matter to Jane. She would consider a raspberry on the bottom with vanilla on top the same as having vanilla under a raspberry dip.

(A) 15 (B) 30 (C) 45 (D) 60 (E) 75 [8] _____

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9. Jessie's math teacher assigned one homework problem Monday night. On each of the following days for the rest of the week, she assigned three times as many problems as on the previous night. At the end of the week, all the problems from the chapter had been assigned. How many problems did the chapter have?

(A) 13 (B) 15 (C) 63 (D) 81 (E) 121 [9] _____

---

10. Which of the following statements is true for the data set below?

$$12, 5, 15, 13, 6, 9, 3, 5, 7, 5$$

(A) mode $<$ median $<$ mean
(B) mode $>$ median $>$ mean
(C) mode $=$ median $=$ mean
(D) mode $<$ mean $<$ median
(E) mode $>$ mean $>$ median [10] _____

Grades 7-8
2007 Math Contest Exam Answer Key

Exam	T1	T2	T3	T4	T5
P1	a	b	a	c	c
P2	e	b	a	c	e
P3	c	a	a	d	e
P4	c	c	c	e	b
P5	e	c	e	c	e
P6	d	a	d	e	c
P7	b	d	b	d	e
P8	c	d	d	d	b
P9	b	d	e	d	e
P10	e	c	c	b	a