

WCTM MATHEMATICS CONTEST, 2004

Test 1

Name: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The distance between $(1, 3)$ and $(4, -1)$ is
 (A) 5 (B) $\sqrt{5}$ (C) $\sqrt{13}$ (D) $\sqrt{2}$ (E) 25 [1] _____

2. The length of the diagonal of the $1 \times 2 \times 1$ cube is
 (A) 5 (B) $\sqrt{5}$ (C) $\sqrt{6}$ (D) 6 (E) 2 [2] _____

3. If x is an angle of a right triangle and $\sin x = \frac{1}{2}$, then $\tan x$ is
 (A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{\sqrt{3}}{3}$ (E) $1 + \sqrt{3}$ [3] _____

4. A store set the selling price of a product 40% above what is cost. When it didn't sell, the price was reduced by 15%. The final price was \$8.65. What did the product cost the store?
 (A) \$9.78 (B) \$6.92 (C) \$8.40 (D) \$6.49 (E) \$7.27 [4] _____

5. Suppose $f(x)$ is a periodic function with the smallest period 2π . If $f(x + \pi) = f(\pi - x)$, then f is
 (A) even (B) odd (C) neither even nor odd (D) both even and odd
 (E) none of these [5] _____

6. How many integers from 992 to 2004 are multiples of 7?
 (A) 286 (B) 287 (C) 145 (D) 146 (E) 141 [6] _____

7. If $a * b = a^2 - b^2 + 1$, then
 (A) $3 * (-1) = 9$ (B) $a * b = b * a$ (C) $1 * a = a$ (D) $a * 0 = 0$ (E) $a * a = a^2$ [7] _____

8. Tom can finish a piece of work in 11 days. After he has worked for 2 days, Mike joins him and together they finish the work in 6 more days. In how many days could Mike finish the work by himself?
 (A) 28 (B) 20 (C) 22 (D) 12 (E) 24 [8] _____

9. Suppose a, b, c are straight lines and β is a plane in space. Then a is parallel to b if

- (A) a and b are parallel to β (B) a and b are perpendicular to c
(C) The angles between a and β and between b and β are equal
(D) a and b are perpendicular to β (E) none of these [9] _____

10. The smallest period of $f(x) = 3 \sin^2 \frac{\pi x}{2} + 1$ is

- (A) 1 (B) 2 (C) 4 (D) 3 (E) none of these [10] _____

WCTM MATHEMATICS CONTEST, 2004

Test 2

NAME: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. If the reciprocal of $x + 2$ is $x - 2$, then x equals
 (A) 2 (B) -2 (C) 2 or -2 (D) $\sqrt{5}$ or $-\sqrt{5}$ (E) 4 or -4 [1] _____

2. If the ratio of the circumference and the area of a circle is $\frac{1}{2}$, then
 (A) the area is 16π (B) the radius is 1 (C) the diameter is 4
 (D) the circumference is 4 (E) none of these [2] _____

3. If A is a set consisting of all integers less than 2004, how many in A are squares of integers?
 (A) 50 (B) 44 (C) 100 (D) 45 (E) 40 [3] _____

4. How many integers n are there satisfying $(n - 1)(n - 2) < 0$?
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 0 [4] _____

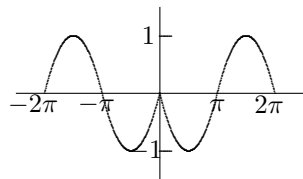
5. The definition domain of function $\sqrt{\ln(3 - 2x)}$ is
 (A) $x > 1$ (B) $x \leq 1$ (C) $x \leq \frac{3}{2}$ (D) $x < \frac{3}{2}$ (E) $x \leq \frac{3+\epsilon}{2}$ [5] _____

6. If x_1 and x_2 are the two roots of $x^2 + ax + b$, then
 (A) $x_1 + x_2 = a$ (B) $x_1x_2 = b$ (C) $|x_1 + x_2| = a$ (D) $x_1x_2 = -b$ (E) none of these [6] _____

7. If A and B are two sets each with 25 elements, the number of elements in $A \cup B$ is:
 (A) 25 (B) 50 (C) 37 (D) 0 (E) depends on the number in $A \cap B$. [7] _____

8. If $x + y = 1$, then $x^2 + y^2$ has
 (A) a maximum value at $x = 1/2$ (B) a minimum value $1/4$ (C) a maximum value 1
 (D) a minimum value $1/2$ (E) none of these [8] _____

9. Which function corresponds to the graph in



- (A) $y = |\sin x|$ (B) $y = \sin |x|$ (C) $y = -|\sin x|$ (D) $y = -\sin |x|$ (E) $y = |\sin |x||$ [9] _____

10. All even numbers greater than -1000 are arranged in six columns in the table below. 2004 appears in which column?

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
-998	-996	-994	-992	-990	-988
-986	-984	-982	-980	-978	-976
-974	-972

- (A) *A* (B) *B* (C) *C* (D) *D* (E) *E* or *F* [10] _____

1. The solution of the inequality $|1 - 2x| < 3$ is
 (A) $x < 3$ (B) $-2 < x < 1$ (C) $-1 \leq x \leq 2$ (D) $-1 < x < 2$ (E) none of these [1] _____

2. The vertex of the parabola $y = 2 - x + x^2$ is
 (A) $(\frac{1}{2}, 0)$ (B) $(0, \frac{7}{4})$ (C) $(-\frac{1}{2}, \frac{7}{4})$ (D) $(\frac{1}{2}, -\frac{7}{4})$ (E) none of these [2] _____

3. How many solutions does the equation $\log_x(4x - 4) = 2$ have?
 (A) 1 (B) 2 (C) 3 (D) infinitely many (E) 0 [3] _____

4. If the diagonal of a square is x , then the area of the square is
 (A) $2x^2$ (B) $4x^2$ (C) x^2 (D) $\frac{x^2}{2}$ (E) $\frac{x^2}{4}$ [4] _____

5. If the system $ax + by = 0$ and $cx + dy = 0$ has only trivial solution $x = 0, y = 0$, then
 (A) $a = b = c = d = 0$ (B) $ad = bc$ (C) the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is not invertible
 (D) the determinant of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is zero
 (E) the determinant of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is not zero [5] _____

6. Let f be the function whose domain is the set of all positive integers defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is an even integer} \\ 2x & \text{if } x \text{ is an odd integer} \end{cases}.$$

Then,

- (A) $f(f(3)) = 6$ (B) $f(4 + f(100)) = 10$ (C) $f(f(f(1))) = 1$
 (D) $f(2f(2)) = 2$ (E) none of these [6] _____

7. If $f(x)$ is a differentiable function in $[-2, 2]$ satisfying $f(-1) = f(1)$, then
 (A) $f(0) = 0$ (B) $f'(0) = 0$ (C) $f'(0) < 0$ (D) $f'(0) > 0$ (E) none of these [7] _____

8. The decimal number $x = 0.2004200420042004 \dots$ can be written as $x = p/q$ where p and q are positive integers without common factors. Then,
(A) $q = 9999$ (B) $p = 167$ (C) $p + q = 4001$ (D) $p - q = 104$ (E) none of these [8] _____
9. The distance between the x and y intercepts of the straight line $x - 2y = 2$ is
(A) 1.5 (B) 3 (C) $\sqrt{3}$ (D) $\sqrt{5}$ (E) none of these [9] _____
10. Use 1, 2, 3, 4, 5 to form 5-digit numbers without repeated digits. How many even numbers less than 50000 can be formed?
(A) 60 (B) 48 (C) 36 (D) 24 (E) none of these [10] _____

1. The number of solutions of $x^2 - 3|x| + 2 = 0$ is
(A) 4 (B) 3 (C) 2 (D) 1 (E) 0 [1] _____
2. The line passing through $(1, -1)$ and $(2, 1)$ is parallel to
(A) $y = x - 3$ (B) $y = 3x - 2$ (C) $y = x - 3$ (D) $y = 2x - 1$
(E) none of these [2] _____
3. What is the coefficient of the imaginary part of the complex number $\frac{(1 - 2i)^2}{1 + i}$?
(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $-\frac{7}{2}$ (D) $\frac{7}{2}$ (E) -1 [3] _____
4. If $a_0 = 0$, $a_1 = 1$ and $a_{n+2} = 3a_n - a_{n+1}$ for $n \geq 0$, then
(A) $a_4 = -5$ (B) $a_4 = -1$ (C) $a_4 = -7$ (D) $a_4 = 4$ (E) none of these [4] _____
5. If $4^n = A$, then
(A) $2^{1+4n} = 2A$ (B) $2^{1+4n} = 2A^2$ (C) $4^{2n+1} = 4A$ (D) $2^n = \frac{A}{2}$
(E) none of these [5] _____
6. The slope of the line tangent to the curve $y = \sin x$ at $x = -\frac{\pi}{4}$ is given by
(A) $-\frac{\sqrt{2}}{2}$ (B) 1 (C) $\frac{\sqrt{2}}{2}$ (D) $1/2$ (E) $-1/2$ [6] _____
7. Which of the following is not always true?
(A) $(x + y)^2 \leq 2(x^2 + y^2)$ (B) $|2x - y| \leq |x| + |x - y|$ (C) $xy \leq \frac{x^2 + y^2}{2}$
(D) $1 + \tan x \geq 1$ (E) $|\sin x| \leq |x|$ [7] _____
8. A person has \$2.75 in dimes and quarters. If she has 3 times as many dimes as quarters, how many coins does she have?
(A) 16 (B) 20 (C) 22 (D) 24 (E) none of these [8] _____

9. A curve is given by the parametric equations: $x = 1 - 3t$, $y = t^2 - t + 2$, $-\infty < t < \infty$. Then,

- (A) the curve passes through the origin
- (B) the curve intersects with y axis
- (C) the curve intersects with x axis
- (D) the curve does not intersect with the straight line $y = x$
- (E) the curve is a circle

[9] _____

10. How many times does the curve $y = \cos(x)$ intersect with the line $y = \frac{100\pi - x}{100\pi}$?

- (A) 50
- (B) 100
- (C) 99
- (D) 200
- (E) none of these

[10] _____

1. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7\}$, then $A \setminus B$, the complement of B relative to A , is
(A) $\{1, 2, 3, -6, -7\}$ (B) $\{1, 2, 3, 0, -6, -7\}$ (C) $\{6, 7\}$ (D) $\{1, 2, 3\}$
(E) none of these [1] _____

2. If \vec{A} and \vec{B} are two perpendicular vectors, then
(A) $\vec{A} + \vec{B} = \vec{0}$ (B) $\vec{A} \cdot \vec{B} = 1$ (C) $\vec{A} = \vec{0}$ or $\vec{B} = \vec{0}$ (D) $\vec{A} \times \vec{B} = \vec{0}$
(E) $\vec{A} \cdot \vec{B} = 0$. [2] _____

3. If A is the constant term in the expansion of $(x^2 - \frac{2}{x})^{100}$, then
(A) $A = \frac{2^{66}99!}{33!66!}$ (B) $A = -\frac{2^{66}99!}{33!66!}$ (C) $A = 0$ (D) $A > 2^{67}$
(E) none of these [3] _____

4. $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{99 \times 101} =$
(A) $\frac{100}{101}$ (B) $\frac{50}{101}$ (C) 1.01 (D) $\frac{1}{2}$ (E) 1 [4] _____

5. A cubic equation $x^3 + ax^2 + bx + c = 0$ with real coefficients has 1 and $1 - i$ as two roots. Then
(A) $a = 4$ (B) $b = -3$ (C) $a + b = 1$ (D) $c = 2$ (E) $c = -1$ [5] _____

6. The rectangular coordinates of point A is $(1, \sqrt{3})$. The polar coordinates (r, θ) for A is
(A) $(1, \frac{\pi}{3})$ (B) $(2, \frac{\pi}{3})$ (C) $(2, \frac{\pi}{6})$ (D) $(2, \sqrt{3})$
(E) $(1, \frac{\sqrt{3}}{2})$ [6] _____

7. $\int_0^1 x\sqrt{1+x^2} dx$ is equal to
(A) $\frac{1}{2} \int_0^1 \sqrt{u} du$ (B) $\int_1^2 \sqrt{u} du$ (C) $\int_0^1 \sqrt{1+u} du$ (D) $\frac{1}{2} \int_0^1 \sqrt{1+u} du$
(E) none of these [7] _____

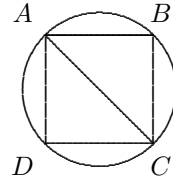
8. $\lim_{n \rightarrow \infty} \frac{-n^2 - n + 17}{3n^2 + 4n - 73} =$

- (A) ∞ (B) $-\infty$ (C) 0 (D) 1 (E) $-\frac{1}{3}$ [8] _____

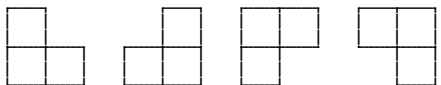
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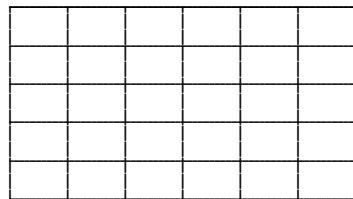
9. In the diagram, $ABCD$ is a square. If the length of AC is 3, find the area of the region lying outside the

square but inside the circle (the center of the circle is on AC).



- (A) $\frac{9}{4}(\pi - 2)$ (B) $\frac{3}{2}(\pi - 1)$ (C) $\frac{9\pi - 3}{8}$ (D) $\frac{\sqrt{3}\pi}{2}$ (E) $\frac{3\pi\sqrt{2}}{5}$ [9] _____

10. If we call any of the shapes  an “L”, how many “L”s are there in



- (A) 80 (B) 20 (C) 40 (D) 60 (E) 100 [10] _____

Grades HS
2004 Math Contest Exam

Questions	Test 1	Test 2	Test 3	Test 4	Test 5
1	a	d	d	a	d
2	c	a	e	d	e
3	d	d	a	b	c
4	e	e	d	c	b
5	a	b	e	b	c
6	c	b	b	c	b
7	a	e	e	d	d
8	c	d	c	b	e
9	d	d	d	b	a
10	b	b	b	d	a