WCTM MATHEMATICS CONTEST, 2003

Test 1 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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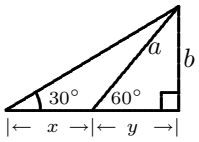
1. The augmented matrix for a system of linear equations reduces to $\left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$. The solution set is:

(A) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + k \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ (B) $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + k \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ (C) $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + k \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

(D) $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + k \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ (E) $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + k \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$

[1] _____

2. 

In the figure

(A) $a = \sqrt{3}\, y$ (B) $a = x$ (C) $a = \dfrac{\sqrt{3}}{2} y$ (D) $a = 2x$ (E) $a = \sqrt{3}\, x$ [2] _____

3. If $F' = e^{2x}$ and $F(0) = 1$ then $F(x) =$

(A) $e^{2x} + 1$ (B) $e^{2x} - 1$ (C) $\dfrac{e^{2x}}{2}$ (D) $\dfrac{e^{2x}}{2} + 1$ (E) $\dfrac{e^{2x}}{2} + \dfrac{1}{2}$ [3] _____

4. $P(x) = x^4 + 3x^2 + 2$ has

(A) 4 real roots (B) 2 distinct real roots and 2 complex roots (C) 4 distinct complex roots

(D) no roots (E) a repeated real root and 2 complex roots

[4] _____

5. A line tangent to the curve $y = \sin x$ at $x = \dfrac{\pi}{3}$ has the equation

(A) $y - \dfrac{\sqrt{3}}{2} = \dfrac{1}{2}\left(x - \dfrac{\pi}{3}\right)$ (B) $y + \dfrac{\sqrt{3}}{2} = \dfrac{1}{2}\left(x - \dfrac{\pi}{3}\right)$ (C) $y + \dfrac{\sqrt{3}}{2} = \left(x - \dfrac{\pi}{3}\right)$

(D) $y - \dfrac{\sqrt{3}}{2} = x + \dfrac{\pi}{3}$ (E) $y + \sqrt{3} = \dfrac{1}{2}\left(x - \dfrac{\pi}{3}\right)$

[5] _____

6. $(-\infty, -3) \cap (-2, \infty) =$

(A) $(-3, -2)$ (B) $(-\infty, \infty)$ (C) $(-\infty, -2)$ (D) $(-\infty, -3)$ (E) the empty set $\emptyset$ [6] _____

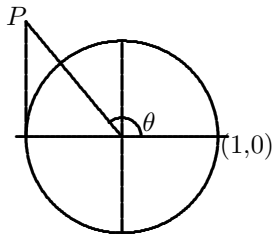
7. If $x^2 - 4x + 3 = 0$ then $x - 2 =$

(A) 0 or 2 (B) -2 or 0 (C) 1 or 2 (D) ± 1 (E) -1 or 2 [7] _____

8. If $z = 3 + 7i$, where $i^2 = -1$ then $\frac{1}{z} =$

(A) $\frac{3}{\sqrt{58}} - \frac{7i}{\sqrt{58}}$ (B) $7 - 3i$ (C) $\frac{3}{58} + \frac{7i}{58}$ (D) $\frac{3}{58} - \frac{7i}{58}$ (E) $\frac{3}{\sqrt{58}} + \frac{7i}{\sqrt{58}}$ [8] _____

9. The coordinates of P are:



(A) $(\cos\theta, \sin\theta)$ (B) $(-\cos\theta, \sin\theta)$ (C) $(\sin\theta, \cos\theta)$ (D) $(-1, \tan\theta)$ (E) $(-1, -\tan\theta)$ [9] _____

10. If $N = (2003)^{2001}$ then the unit digit of N is

(A) 3 (B) 9 (C) 1 (D) 7 (E) 2 [10] _____

Test 2 A/B *Name*: _____ *School*: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. $1 + 23 + 45 + 67 + \cdots + 2003 =$

   (A) 92140 (B) 92162 (C) 92184 (D) $184,368$ (E) $184,324$ [1] _____

2. The graph of $y = 3x^4 - x^3$ is increasing when

   (A) $x < 1$ (B) $x > 1$ (C) $x > \frac{1}{4}$ (D) $x < \frac{1}{4}$ (E) $x < 4$ [2] _____

3. $3 + 3\left(\frac{5}{3}\right) + 3\left(\frac{5}{3}\right)^2 \cdots + 3\left(\frac{5}{3}\right)^n + \cdots =$

   (A) $\frac{15}{2}$ (B) 5 (C) 15 (D) 6 (E) none of these [3] _____

4. If $A = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then

   (A) $a = 1$ (B) $a = 6$ (C) $a = \frac{1}{2}$ (D) $a = -2$ (E) A^{-1} does not exist [4] _____

5. If $|2 - 5x| > 4$ then

   (A) $x > -\frac{2}{5}$ (B) $x < -\frac{2}{5}$ (C) $x < -\frac{2}{5}$ or $x > \frac{6}{5}$ (D) $x > \frac{6}{5}$ (E) $-\frac{2}{5} < x < \frac{6}{5}$ [5] _____

6. If $f(x) = \sqrt{x}$ then

   (A) $f^{-1}(x)$ does not exist (B) $f^{-1}(x) = \sqrt{x}$ (C) $f^{-1}(x) = \frac{1}{\sqrt{x}}$

   (D) $f^{-1}(x) = \frac{1}{x^2}, x \geq 0$ (E) $f^{-1}(x) = x^2, x \geq 0$ [6] _____

7. In base 10, $(2003)_{(\text{base } 7)} - (1066)_{(\text{base } 7)}$ is

   (A) 937 (B) 604 (C) 634 (D) 9310 (E) 298 [7] _____

8. Four women and three men are on the City Council of Erehwon. A mayor and vice-mayor are selected. The mayor is a woman and the vice-mayor is a man. How many ways can this be done?

(A) 6 (B) 12 (C) 7 (D) $\frac{4!3!}{7!}$ (E) none of these [8] _____

9. $\cos 15^\circ$ is exactly

(A) $\frac{\sqrt{6}-\sqrt{2}}{4}$ (B) $\frac{\sqrt{6}-1}{4}$ (C) $\frac{\sqrt{3}+\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}-\sqrt{2}}{2}$ (E) $\frac{\sqrt{6}+\sqrt{2}}{4}$ [9] _____

10. If $u = (3, 4, 5)$ and $v = (3, -1, z)$ then $u \perp v$ if and only if

(A) $z = 2$ (B) $z = 1$ (C) $z = -1$ (D) $z = -2$ (E) $z = 0$ [10] _____

Test 3 A/B *Name*: _____ *School*: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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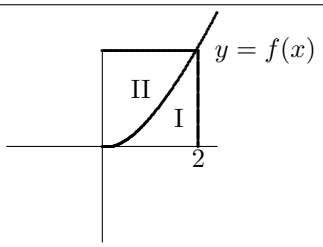
1. If $f(x) = 2x^2$ the ratio of $\dfrac{\text{Area I}}{\text{Area II}}$ is



(A) $\dfrac{2}{3}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{1}{4}$ (E) $\dfrac{1}{2}$ [1] _____

2. The number of subsets of a set with 23 elements is

(A) 23 (B) 2^{23} (C) $\dfrac{2^{23}}{2}$ (D) $2^{23} - 1$ (E) none of these [2] _____

3. Given a geometric sequence $a, ar, ar^2 \cdots$, if $a = 2$ and $r = 3$ then the 20th term is

(A) 3^{20} (B) $2 \cdot 3^{20}$ (C) 3^{19} (D) $2 \cdot 3^{19}$ (E) 6^{19} [3] _____

4. The smallest integer that can be written in the form $30a + 21b$, where a, b are integers is

(A) 2 (B) gcd(30,21) (C) 6 (D) 9 (E) 1 [4] _____

5.

A	B	C	D	E	F	G	H
		1	2	3	4	5	6
7	8	9	10	11	12	·	·
·	·	·	·	·	·	·	·

2003 is in column

(A) A (B) B (C) C (D) D (E) none of these [5] _____

6. $\lim\limits_{x \to 1} \dfrac{e^x + 1}{e^x - 1}$ is

(A) 2 (B) $\dfrac{1}{2}$ (C) not defined (D) between 2 and 3 (E) finite but none of these [6] _____

7. If $y = b^x$, where $b > 0$ then $x =$

(A) $\ln y$ (B) $\dfrac{\ln y}{\ln b}$ (C) $\dfrac{\ln b}{\ln y}$ (D) $\ln b - \ln y$ (E) $-\ln y - \ln b$ [7] _____

8. The coefficient of x^6y^{28} in the expansion of $(x^2 - 2y^7)^7$ is

(A) 560 (B) -560 (C) 35 (D) (-35) (E) none of these [8] _____

9. If $y = 3\cos\frac{\pi}{3}x$ is periodic with period p and amplitude a, then $a + p =$

(A) 6 (B) 3 (C) 9 (D) $6 + \pi$ (E) $3 + \frac{\pi}{2}$ [9] _____

10. When integer N is divided by 7 the remainder is 3. When N is divided by 12 the remainder is 2. The smallest positive value for N is

(A) < 30 (B) between 30 and 36 (C) between 36 and 42
(D) between 42 and 54 (E) greater than 54

[10] _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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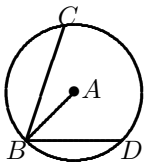
1. In the figure if $\overset{\frown}{BC}= \frac{\pi}{3}, \overset{\frown}{BD}= \frac{\pi}{3}$, the radius $|AB| = 1$, what does $\angle CBD =$



(A) $\frac{4\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{4}$ [1] _____

2. The coordinates of P and Q are $(1, a)$ and $(a, -1)$ respectively, where $a \neq 0$. If O has coordinates (0,0) then

(A) length of $OP = 1 + a^2$ (B) length of $PQ = 1 + a^2$ (C) $OP \perp OQ$

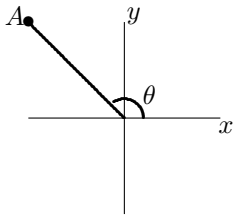
(D) $OP \parallel OQ$ (E) none of the statements is true

[2] _____

3. Four boys and three girls stand in a row. If the boys must all stand together, how many ways are there for the boys and girls to stand?

(A) 144 (B) 3720 (C) 576 (D) 332 (E) 288 [3] _____

4. If the coordinates of A are (x, y) than $\tan\theta =$



(A) $\frac{x}{y}$ (B) $\frac{y}{x}$ (C) $-\frac{x}{y}$ (D) $-\frac{y}{x}$ (E) $-\frac{y}{\sqrt{x^2+y^2}}$ [4] _____

5. When the positive integer N is divided by 5 the remainder is 3. When N is divided by 9 the remainder is 7. When N is divided by 2 the remainder is 0. The smallest positive value for N is

(A) < 80 (B) > 80 and < 90 (C) > 90 and < 120 (D) > 120 and < 150 (E) > 150 [5] _____

6. The distance from (2,3) to the line $y = x$ is

(A) 3 (B) $\sqrt{2}$ (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) 2 [6] _____

7. A $2'' \times 2'' \times 2''$ block of wood weighs 8 oz. A circular hole 1 in. in diameter perpendicular to the face of the cube is drilled through the cube. The block now weighs

(A) $\frac{\pi}{2}$ oz. (B) $(8-\pi)$ oz. (C) $\left(8-\frac{\pi}{2}\right)$ oz. (D) $\left(4-\frac{\pi}{2}\right)$ oz. (E) none of these [7] _____

8. If $z \neq 0$ is complex and $\frac{z-2}{z}$ is a pure imaginary number, then

(A) z is a pure imaginary number (B) $|z+i|=1$ (C) $|z-i|=1$
(D) $|z-1|=1$ (E) $|z+1|=1$

[8] _____

9. Two balls are drawn from an urn containing 3 red and 4 white balls. The first ball is <u>not</u> replaced in the urn before the second ball is drawn. The probability that the second ball is red is

(A) $\frac{4}{7}$ (B) $\frac{3}{6}$ (C) $\frac{3}{7}$ (D) $\frac{3}{12}$ (E) $\frac{4}{12}$ [9] _____

10. If $y = 3\sin\theta - \cos\theta$ then $y'' + y =$

(A) $3(\cos\theta + \sin\theta) - (\sin\theta + \cos\theta)$ (B) $4\sin\theta - \cos\theta$ (C) 0 (D) $2\sin\theta$ (E) $\sin 2\theta$ [10] _____

Test 5 A/B *Name*: _____ *School*: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. If $A = \{1, 3, a\}, B = \left\{1, a^2\right\}$ and $A \cup B = \{1, 3, a\}$, then the number of possible real numbers a is

   (A) 1 (B) 2 (C) 5 (D) 4 (E) none of these [1] _____

2. If real numbers x, y satisfy $\frac{x^2}{4} + y^2 = x$, then $x^2 + y^2$ has

   (A) a minimum value of $-\frac{1}{3}$, but no maximum value
   (B) a minimum value of $-\frac{1}{3}$, and a maximum value of 16
   (C) a minimum value of 0, but no maximum value
   (D) a minimum value of 0, and a maximum value of 16
   (E) none of these

   [2] _____

3. The solution set of the inquality $\ln(x^2) < (\ln x)^2$ is

   (A) $\left(\frac{1}{e^2}, 1\right)$ (B) $\left(e^2, \infty\right)$ (C) $\left(\frac{1}{e^2}, 1\right) \cup \left(e^2, \infty\right)$ (D) $(0, 1) \cup \left(e^2, \infty\right)$ (E) none of these [3] _____

4. If $\frac{1}{e} < a < b < 1$, which of the following is correct?

   (A) $a^a < a^b$ (B) $b^a < b^b$ (C) $a^a < b^a$ (D) $b^b < a^a$ (E) none of these [4] _____

5. If an even function $f(x)$ is periodic with period 2, and $f(x) = x - 1$ when $x \in [1, 2]$, then $f(-9.9) =$

   (A) 0.1 (B) -0.9 (C) 0.9 (D) -0.1 (E) -10.9 [5] _____

6. If the straight lines $(3 - a)x + (2a - 1)y + 7 = 0$ and $(2a + 1)x + (a + 5)y - 6 = 0$ are perpendicular to each other then $a =$

   (A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{7}$ (E) $\frac{1}{6}$ [6] _____

7. The smallest period of $y = \frac{\tan x}{1 - \tan^2 x}$ is

   (A) 2π (B) π (C) $\frac{\pi}{2}$ (D) no period (E) none of these [7] _____

8. The remainder of 2003^{2003} divided by 6 is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 [8] _____

9. Let $f(x) = x^2 + x + a$ with $a > 0$. If $f(y) < 0$, then $f(1 + y)$ is

(A) positive (B) negative (C) nonnegative (D) nonpositive (E) undetermined [9] _____

10. Which set can be parametric equations for $xy = 1$?

(A) $\begin{cases} x = \sqrt{t} \\ y = \frac{1}{\sqrt{t}} \end{cases}$ (B) $\begin{cases} x = \sin t \\ y = \frac{1}{\sin t} \end{cases}$ (C) $\begin{cases} x = \cos t \\ y = \frac{1}{\cos t} \end{cases}$

(D) $\begin{cases} x = \tan t \\ y = \frac{1}{\tan t} \end{cases}$ (E) any of these

[10] _____

Grades HS
2003 Math Contest Exam

Exam	T1	T2	T3	T4	T5
P1	d	c	e	b	b
P2	b	c	b	c	d
P3	e	e	d	c	d
P4	c	e	b	b	c
P5	a	c	e	b	c
P6	e	e	d	d	d
P7	d	e	b	c	c
P8	d	b	a	d	e
P9	e	e	c	c	a
P10	a	c	c	c	d