WCTM MATHEMATICS CONTEST, 2002

Test 1 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. $\frac{3}{2}+\frac{6}{2}+\frac{9}{2}+\cdots+150=$

   (A) 7650 (B) 7575 (C) 15,300 (D) 15,150 (E) 30,300 [1] _____

2. If $f(x)=\frac{x-2}{(x^2+3)(x-4)}$ then $f(x)>0$ for:

   (A) $x<4$ (B) $2<x<4$ (C) $x<2$ or $x>12$ (D) $x>3$ (E) $x>4$ or $x<2$ [2] _____

3. Angle x is in quadrant 1, $\sin 2x=\frac{15}{17}, \cos x=\frac{5}{\sqrt{34}}$, then $\sin x=$

   (A) $\frac{3}{5}$ (B) $\frac{3}{\sqrt{34}}$ (C) $\frac{5}{\sqrt{34}}$ (D) $\frac{5}{3}$ (E) $\frac{3}{2\sqrt{34}}$ [3] _____

4. The number of different arrangements of the letters **AARDVARK** is:

   (A) 8! (B) $\frac{8!}{3!5!}$ (C) 5! (D) $\frac{8!}{5!}$ (E) $\frac{8!}{3!2!}$ [4] _____

5. $f(x)=\frac{x^2-6x+8}{x^2+x-6}$ is continuous for all x except:

   (A) $x=2$ and $x=3$ (B) $x=-2$ and $x=3$ (C) $x=2$ and $x=-3$ (D) $x=-3$ (E) $x=0$ [5] _____

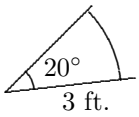
6. $\lim_{x\to 2}\left(\frac{x^2-6x+8}{x^2+x-6}\right)=$

   (A) $-\frac{2}{5}$ (B) not defined (C) $\frac{2}{5}$ (D) 0 (E) $-\frac{4}{3}$ [6] _____

7. Two balls are drawn from a jar, without replacement. The jar has 2 red and 3 blue balls. The probability that the second ball is blue is:

   (A) greater than the probability the first is blue
   (B) equal to the probability the first is blue
   (C) $\frac{2}{4}$
   (D) $\frac{4}{5}$
   (E) none of these

   [7] _____

8. The area of the sector of a circle is: 

(A) 90ft^2 (B) $90\pi\text{ft}^2$ (C) $\frac{3\pi}{20}\text{ft}^2$ (D) $\frac{\pi}{2}\text{ft}^2$ (E) 150ft^2 [8] _____

9. One 1-dollar bill, two 2-dollar bills, five 5-dollar bills and ten 10-dollar bills are in a jar. A single bill is to be selected from the jar. The "expected value" of the experiment is: (rounded to the nearest cent)

(A) \$10 (B) \$5 (C) \$7.22 (D) \$9.32 (E) none of these [9] _____

10. The product of the roots of $p(x) = x^2 - 6x + 8 = 0$ is:

(A) -6 (B) -3 (C) 0 (D) 8 (E) -8 [10] _____

Test 2 A/B *Name*: _____ *School*: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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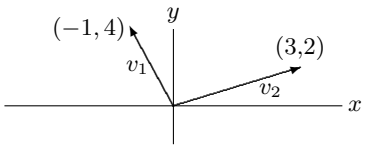
1. v_1 and v_2 are vectors as shown. $2v_1 + 3v_2 =$

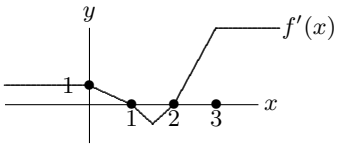


(A) $(2, 6)$ (B) $(4, 6)$ (C) 5 (D) $(7, 14)$ (E) $(11, 14)$ [1] _____

2. For the vectors in problem 1, $|v_1 + v_2| =$

(A) $2\sqrt{10}$ (B) $\sqrt{42}$ (C) 8 (D) $\sqrt{8}$ (E) $\sqrt{15} + \sqrt{13}$ [2] _____

3. The graph shown is the graph of $f'(x)$. The graph of $f(x)$ is concave up if



(A) $x < 1$ or $x > 2$ (B) $x > \frac{3}{2}$ (C) $\frac{3}{2} < x < 3$ (D) $1 < x < 2$ (E) $x < \frac{3}{2}$ or $x > 3$ [3] _____

4. For the graph in problem 3, f has a horizontal tangent line if:

(A) $x = \frac{1}{2}$ and $x = 3$ (B) $x = \frac{3}{2}$ (C) $x < 1$ and $x > 3$ (D) $x = 1$ only (E) $x = 1$ and $x = 2$ [4] _____

5. If A and B are in the second quadrant, $\sin A = \frac{3}{5}$ and $\cos B = -\frac{5}{13}$, then $\cos(A + B) =$

(A) $\frac{63}{65}$ (B) $\frac{33}{65}$ (C) $-\frac{63}{65}$ (D) $-\frac{33}{65}$ (E) $-\frac{16}{65}$ [5] _____

6. $P(x) = 2x^5 - 3x^4 + 2x^3 + x - 1$ has

(A) no real roots
(B) no negative roots
(C) two negative and one positive root [6] _____
(D) exactly two positive roots
(E) one negative and two positive roots

7. If $\dfrac{A}{x+3}+\dfrac{B}{x-2}=\dfrac{5x+5}{(x+3)(x-2)}$, then

(A) $A<B$ (B) $A=B$ (C) there is no solution for A and B (D) $A=-B$ (E) $B-A=5$ [7] _____

8. If $x<-2$ and $x<5$, then

(A) $-2<x<5$ (B) $x<5$ (C) $x<-2$ (D) $-5<x<2$ (E) $-5<x<-2$ [8] _____

9. If $u=\cos\theta$ and $\pi<\theta<\dfrac{3\pi}{2}$, then $\tan\theta=$

(A) $\dfrac{-u}{\sqrt{1+u^2}}$ (B) $\dfrac{-u}{1+u^2}$ (C) $\dfrac{-u}{\sqrt{1-u^2}}$ (D) $\dfrac{-\sqrt{1+u^2}}{u}$ (E) $\dfrac{-\sqrt{1-u^2}}{u}$ [9] _____

10. $2+\dfrac{1}{2}+\dfrac{1}{8}+\dfrac{1}{32}\cdots=$

(A) $\dfrac{8}{3}$ (B) $\dfrac{85}{32}$ (C) 2.5 (D) greater than 3 but finite (E) infinite [10] _____

Test 3 A/B *Name*: _____ *School*: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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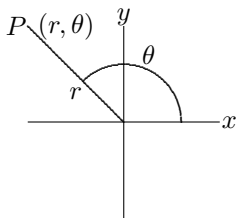
1. The point P has polar coordinates (r, θ). The x, y coordinates for P are



(A) $(r\sin\theta, r\cos\theta)$ (B) $(-r\sin\theta, r\cos\theta)$ (C) $\sqrt{r}\sin\theta, \sqrt{r}\cos\theta)$

(D) $(r\cos\theta, r\sin\theta)$ (E) $(-r\cos\theta, r\sin\theta)$

[1] _____

2. $(\ln x)^2 + 3\ln x - 4 = 0$ has

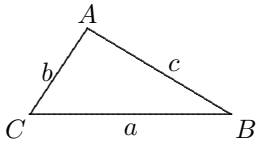
(A) two real solutions
(B) one positive solution
(C) two solutions—one real, one complex
(D) two complex solutions
(E) one negative solution

[2] _____

3. r and s are functions of time t, and $r = s^2$. Also, $\dfrac{ds}{dt} = 3$ when $s = 6$. So $\dfrac{dr}{dt} =$ ____ at time $s = 6$?

(A) 108 (B) 18 (C) 36 (D) 59 (E) 9 [3] _____

4. If $\cos A = \dfrac{3}{5}$, $b = 7$, $c = 10$, then $a =$



(A) $\sqrt{65}$ (B) $\sqrt{86}$ (C) $\sqrt{233}$ (D) $\sqrt{107}$ (E) $\sqrt{190}$ [4] _____

5. If $i^2 = -1$ and $z = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$, then $z^5 =$

(A) $16\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$ (B) $32\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ (C) $32\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

(D) $32\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$ (E) $32\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

[5] _____

6. If $|4 + 3x| > 5$ then

(A) $x < \frac{1}{3}$ (B) $x > -3$ (C) $x > \frac{1}{3}$ or $x < -3$ (D) $-3 < x < \frac{1}{3}$ (E) $\frac{1}{3} < x < -3$ [6] _____

7. $\lim_{x\to\infty} \frac{3\sin x}{x^3 - 3x + 4} =$

(A) 3 (B) 0 (C) $+\infty$ (D) $-\infty$ (E) does not exist [7] _____

8. The amplitude of $y = -3\sin(2x + 4)$ is

(A) less than 2 (B) between 2 and 3 (C) -3 (D) 3 (E) 6 [8] _____

9. The equation of a circle with center $(-2, 3)$ and radius 4 is

(A) $(x-2)^2 + (y-3)^2 = 4$ (B) $(x+2)^2 + (y-3)^2 = 4$ (C) $(x+2)^2 + (y-3)^2 = 16$

(D) $(x-2)^2 + (y-3)^2 = 16$ (E) $x^2 + 2x + y^2 - 9y = 16$

[9] _____

10. The exact value of $\sin(15°)$ is

(A) $\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$ (B) $2\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$ (C) $-\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$ (D) $\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$ (E) $-\sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}}$ [10] _____

Test 4 A/B *Name*: _____ *School*: _____

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1. The inverse matrix $\begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 7 \end{pmatrix}$ has _____ in the upper right-hand corner.

(A) 1 (B) $\frac{3}{14}$ (C) -1 (D) $-\frac{3}{14}$ (E) $\frac{3}{2}$ [1] _____

2. $\begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} =$

(A) $(2, 7, 8)$ (B) 19 (C) $\begin{pmatrix} 2 \\ 7 \\ 8 \end{pmatrix}$ (D) $\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$ (E) $(2, 4, 8)$ [2] _____

3. The number of bacteria in a culture at time t is $N = N_0 e^{3t}$ where N_0 is the number at time $t = 0$. How long does it take to double the number of bacteria in the culture?

(A) $\frac{1}{3}\ln 5$ (B) $\ln 8$ (C) 2 (D) $\frac{1}{2}$ (E) none of these [3] _____

4. The multiplicative inverse of the complex number $2 + 3i$ is

(A) $\frac{2-3i}{13}$ (B) $3 + 2i$ (C) $3 - 2i$ (D) $2 - 3i$ (E) $\frac{2-3i}{\sqrt{13}}$ [4] _____

5. The tangent line to the circle $x^2 + y^2 = 4$ at $\left(1, -\sqrt{3}\right)$ has slope

(A) $-\sqrt{3}$ (B) $\sqrt{3}$ (C) $-\frac{\sqrt{3}}{3}$ (D) $\frac{\sqrt{3}}{3}$ (E) $-\frac{1}{4}$ [5] _____

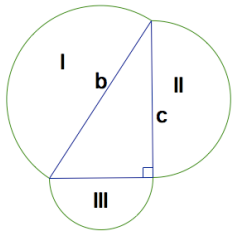
6. The greatest common divisor of pq^3r^2, $p^2q^2r^2$ and p^3q^4r where p, q and r are prime numbers is

(A) pqr (B) $p^3q^4r^2$ (C) pq^2r (D) 1 (E) p^2q^3r [6] _____

7. April 1 was Monday this year. In the year 2012 April 1 will be on a _____ . Remember: Three were leap years 2004, 2008, 2012.

(A) Friday (B) Sunday (C) Monday (D) Wednesday (E) Tuesday [7] _____

8. The right triangle ABC has semicircles constructed with diameters a, b and c with areas I, II and III, as shown below. Which of the statements is true?



(A) area I + area II + area III $= \dfrac{ab}{2}$ (B) area I = area II + area III

(C) $(\text{area I})^2 = (\text{area II})^2 + (\text{area III})^2$ (D) $\sqrt{\text{area I}} = \sqrt{\text{area II}} + \sqrt{\text{area III}}$

(E) more information is required to get a formula [8] _____

9. If $\ln y = 3x$ then $y =$

(A) $3 \ln x$ (B) $3e^x$ (C) x^3 (D) e^{3x} (E) none of these [9] _____

10. The distance between the parallel lines $y = 2x$ and $y = 2x + 5$ is

(A) 4 (B) 2 (C) $\dfrac{5}{2}$ (D) 5 (E) $-\sqrt{5}$ [10] _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. If 215_8 means the base is 8, then $(215_8)\,(26_8) =$

   (A) 5590_8 (B) 5631_8 (C) 6036_8 (D) 6026_8 (E) none of these [1] _____

2. A large stone statue weighs 700 lbs. A smaller scale model of the statue is made of the same material, and the scale is 1:10. The weight of the smaller model is

   (A) 70 lbs (B) 7 lbs (C) 0.7 lbs (D) 350 lbs (E) 35 lbs [2] _____

3. Five numbers are selected without replacement from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. There are _____ possible different sets.

   (A) $9!$ (B) $5!$ (C) $\dfrac{9!}{5!}$ (D) $\dfrac{9!}{4!}$ (E) $\dfrac{9!}{5!\,4!}$ [3] _____

4. The equation of a line through $(-2, 3)$ and parallel to $3y + x = 5$ is

   (A) $3y + x = 7$ (B) $3y + x = 4$ (C) $3y + x = 11$ (D) $y - 3x = 15$ (E) none of these [4] _____

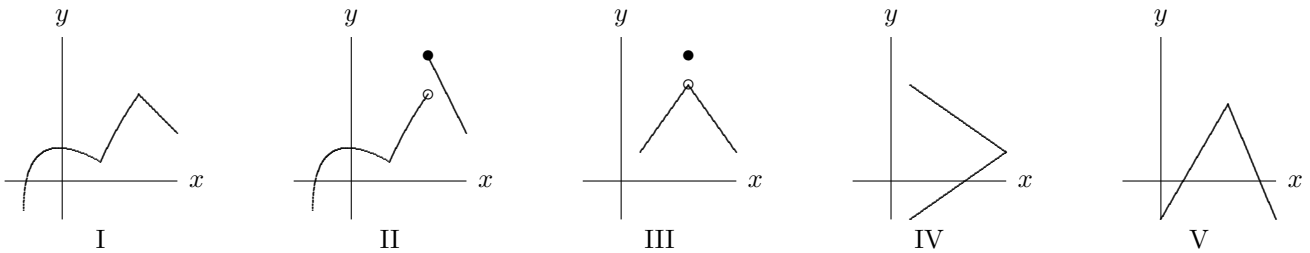
5. The equation of a line parallel to the line through (a, b) and $(2a, 3b)$ is

   (A) $ay = bx$ (B) $3ay = 4bx$ (C) $4by = 3ax$ (D) $ay = 2bx$ (E) $2ay = bx$ [5] _____

6. If $\tan x = \dfrac{3}{4}$ and $\sin x < 0$ then $x =$

   (A) $\arctan \dfrac{3}{4}$ (B) $\pi + \arctan \dfrac{3}{4}$ (C) $\dfrac{\pi}{2} + \arctan \dfrac{3}{4}$ (D) $-\dfrac{\pi}{2} + \arctan \dfrac{3}{4}$ (E) $\arctan\left(\tan \dfrac{3}{4}\right)$ [6] _____

7. For which graph is y not a function of x ?



   (A) I (B) II (C) III (D) IV (E) V [7] _____

8. If $ax^2 + bx + c = 0$ has two roots and the difference of the roots is $-\frac{b}{a}$, then

(A) $b^2 = -4ac$ (B) $b = 0$ (C) $b^2 = 4ac$ (D) $a = 0$ (E) $c = 0$ [8] _____

9. $y^2 = -3x + 6$ is the equation of

(A) an ellipse through $(2, 0)$
(B) a parabola opening up and through $(2, 0)$
(C) a parabola opening down and through $(2, 0)$
(D) a parabola opening to the right and through $(2, 0)$
(E) a parabola opening to the left and through $(2, 0)$

[9] _____

10. The area bounded by the x-axis, the y-axis and the curve $y = -x^2 + 16$ is shown in Figure A. Approximate this area with the four trapezoids in Figure B.

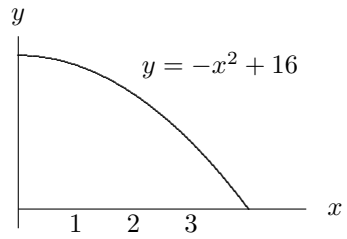


Fig A

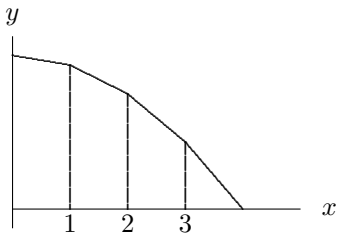


Fig B

(A) 42 (B) $\frac{128}{3}$ (C) 39 (D) 41 (E) 43 [10] _____

Exam	T1	T2	T3	T4	T5
P1	b	d	d	b	c
P2	e	a	a	c	c
P3	b	c	c	e	e
P4	e	e	a	a	a
P5	c	e	c	d	d
P6	a	b	c	c	b
P7	b	a	b	b	d
P8	d	c	d	b	e
P9	c	e	c	d	e
P10	d	a	a	e	a