

WCTM MATHEMATICS CONTEST, 2001

Test 1

NAME: \_\_\_\_\_

CLASS AB

SCHOOL: \_\_\_\_\_

SCORING: 20 points for each correct answer, -5 for each wrong answer.

---

1.  $y = 5e^{-\frac{t}{3}}$  is the equation of radioactive decay of a particular isotope in years ( $t$ ). At time  $t = 0$   $y = 5$ . When is  $y = 2.5$ ?

- (A) 3            (B)  $\frac{3}{2}$             (C)  $3 \ln 2$             (D)  $-3 \ln 2$             (E) none of these            [1] \_\_\_\_\_

2.  $-2 + 2 + 6 + \dots + 902 =$

- (A) (225)(450)    (B) (226)(450)    (C) (225)(900)    (D) (226)(900)    (E) (227)(450)    [2] \_\_\_\_\_

3. The number of arrangements of the letters **BOOKKEEPER** is

- (A)  $10!$             (B)  $\frac{10!}{2!2!3!}$             (C)  $\frac{10!}{3!}$             (D)  $\frac{10!}{2!}$             (E)  $\frac{10!}{2!2!3!3!}$             [3] \_\_\_\_\_

4. The gcd of 2872 and 1296 is in the interval

- (A) [1, 4]            (B) [4, 7]            (C) [8, 11]            (D) [11, 14]            (E) [14, 17]            [4] \_\_\_\_\_

5.  $\ln_{base 7} (131)(65)$  is

- (A) 12,505            (B) 22,975            (C) 240            (D) 2164            (E) none of these            [5] \_\_\_\_\_

6. The distance between the line  $y = 2x$  and the point  $(1, -3)$  is:

- (A)  $\sqrt{5}$             (B) 5            (C) 3            (D)  $\sqrt{3}$             (E) 4            [6] \_\_\_\_\_

7. The distance between the points  $(3,5)$  and  $(-4, 2)$  is

- (A)  $\sqrt{10}$             (B)  $\sqrt{8}$             (C) 8            (D)  $\sqrt{58}$             (E) 58            [7] \_\_\_\_\_

8. The equation of the line through  $(1, -1)$  perpendicular to  $y = 2x + 7$  is

- (A)  $y = -x$     (B)  $y = \frac{1}{2}x - 1$     (C)  $y = \frac{1}{2}x + \frac{1}{2}$     (D)  $y = -\frac{1}{2}x - \frac{1}{2}$     (E) none of these    [8] \_\_\_\_\_

9. The altitude of an equilateral triangle with sides 3 inches long is

- (A) 3 inches      (B)  $\frac{\sqrt{3}}{2}$  inches      (C)  $3\sqrt{3}$  inches      (D)  $\frac{3\sqrt{3}}{2}$  inches      (E)  $\left(\frac{3}{2}\right)$  inches      [9] \_\_\_\_\_

10. A solid block of salt weighs 100 grams. A second block of the same kind of salt is similar to the first with each corresponding side  $k$  times as long. The weight of the second block is

- (A)  $10k^2$  grams      (B)  $k$  grams      (C)  $100k^3$  grams      (D)  $100k$  grams      (E)  $100k^2$  grams      [10] \_\_\_\_\_

**SCORING:** 20 points for each correct answer, -5 for each wrong answer.

---

1. The area of the region bound by  $y = x$  and  $y = x^2$  is:

- (A)  $\frac{2}{3}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{6}$       (D)  $\frac{1}{2}$       (E)  $\frac{1}{4}$       [1] \_\_\_\_\_

2. If  $p$  is the number of bacteria in a culture jar at time  $t$  and  $p = p_0 e^{-.3t}$ , where  $p_0$  is the number present at time  $t = 0$ , then at what time is the population  $\frac{1}{2}$  of the original population?

- (A)  $t = \frac{10}{3}$       (B)  $t = \frac{\ln 2}{.3}$       (C)  $t = .3 \ln 2$       (D)  $t = \frac{.3}{\ln 2}$       (E)  $t = 3 \ln 2$       [2] \_\_\_\_\_

3. The present value  $B$  of a bank deposit using periodic compounding at periodic rate  $r$  and  $n$  time intervals is  $B = \frac{A}{(1+r)^n}$  where  $A$  is the value at time 0. The present value of a deposit which will yield \$1,000 five years from now at 6% per year compounded semiannually is:

- (A)  $\frac{1000}{(1.06)^5}$       (B)  $\frac{1000}{(1.03)^5}$       (C)  $\frac{1000}{(1.03)^{10}}$       (D)  $\frac{1000}{(1.12)^5}$       (E)  $\frac{1000}{(1.12)^{10}}$       [3] \_\_\_\_\_

4. If demand and supply are respectively  $p(x) = \frac{185-x}{35}$  and  $s(x) = \frac{x+40}{70}$  then supply equals demand when  $x =$

- (A) 100      (B) 220      (C) 300      (D) 150      (E) 110      [4] \_\_\_\_\_

5. Three red balls and one white ball are in a jar. Three balls are drawn without replacement. The probability that at least one of the balls drawn is red is:

- (A)  $\frac{1}{4}$       (B)  $\frac{3}{4}$       (C) 1      (D)  $\frac{27}{32}$       (E) none of these      [5] \_\_\_\_\_

6. A co-ed basketball team has four boys and one girl. For a team picture they all sit in one row. The probability the girl is sitting between two boys is:

- (A)  $\frac{2}{5}$       (B)  $\frac{1}{5}$       (C)  $\frac{3}{4}$       (D)  $\frac{3}{5}$       (E)  $\frac{1}{4}$       [6] \_\_\_\_\_

7. Three red and two blue balls are in an urn. Two balls are drawn without replacement. The probability the second ball drawn is blue is:

- (A)  $\frac{3}{10}$       (B)  $\frac{1}{10}$       (C)  $\frac{3}{9}$       (D)  $\frac{7}{10}$       (E)  $\frac{2}{5}$       [7] \_\_\_\_\_

8. In the problem above, the probability the second ball is blue given the first ball is red is:

- (A)  $\frac{3}{10}$       (B)  $\frac{1}{2}$       (C)  $\frac{3}{5}$       (D)  $\frac{1}{10}$       (E)  $\frac{2}{5}$       [8] \_\_\_\_\_

9. How many 4-digit house numbers are there with exactly one zero (not on the left) and no repeats?

- (A)  $9!$       (B)  $\frac{9!}{5!}$       (C) 1512      (D)  $\frac{10!}{5!}$       (E)  $\frac{10!}{6!}$       [9] \_\_\_\_\_

10. There are \_\_\_\_\_ 3-element subsets of a 5-element set?

- (A)  $3!$       (B)  $\frac{5!}{3!}$       (C)  $5!$       (D)  $\frac{5!}{3!2!}$       (E)  $\frac{5!}{2!}$       [10] \_\_\_\_\_

**SCORING:** 20 points for each correct answer, -5 for each wrong answer.

---

1. In the Olympic Hockey tournament the U.S. played six games. They won three games, lost two and tied one. This could happen in how many ways?

- (A) 10                      (B) 15                      (C) 20                      (D) 30                      (E) 60                      [1] \_\_\_\_\_

2. The third entry in row one of the inverse of  $A$  is \_\_\_\_\_, where  $A = \begin{pmatrix} 6 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 7 \end{pmatrix}$

- (A)  $\frac{2}{3}$                       (B)  $-\frac{2}{3}$                       (C) 0                      (D) -1                      (E) 1                      [2] \_\_\_\_\_

3. If  $T(a) = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & -1 \\ 1 & 1 & 2 \end{pmatrix} (a)$ , then  $T \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} =$

- (A) 10                      (B)  $(12, -6, 5)$                       (C)  $\begin{pmatrix} 12 \\ -6 \\ 5 \end{pmatrix}$                       (D)  $\begin{pmatrix} 10 \\ -6 \\ 6 \end{pmatrix}$                       (E)  $(10, -6, 6)$                       [3] \_\_\_\_\_

4. If  $A$  is an  $m \times n$  matrix and  $B$  is  $n \times m$ , then  $AB = BA$

- (A) always                      (B) never                      (C) sometimes                      (D) when  $m = n$                       (E) when  $m < n$                       [4] \_\_\_\_\_

5.  $\frac{a}{x-2} + \frac{b}{x+3} = \frac{3x+4}{(x-2)(x+3)}$ .  $a =$

- (A) -2                      (B) 2                      (C) 1                      (D) -1                      (E) 3                      [5] \_\_\_\_\_

6. The sum of the roots of  $p(x) = 3x^2 - 11x + 6$  is:

- (A) -11                      (B) 11                      (C)  $-\frac{11}{3}$                       (D)  $\frac{11}{3}$                       (E) none of these                      [6] \_\_\_\_\_

7. The polynomial  $p(x) = 3x^4 - 2x^2 + 7x - 3$  has

- (A) 0 real roots                      (B) 1 positive and 1 negative real root                      (C) 2 positive real roots  
(D) 2 negative real roots                      (E) no real roots

[7] \_\_\_\_\_

8. If  $u = \tan \theta$  and  $0 \leq \theta \leq \frac{\pi}{2}$  then  $\sin \theta =$

- (A)  $\sqrt{1+u^2}$       (B)  $\frac{1}{\sqrt{1+u^2}}$       (C)  $\frac{u}{\sqrt{1+u^2}}$       (D)  $\sqrt{u^2-1}$       (E)  $\frac{1}{\sqrt{u^2}}$       [8] \_\_\_\_\_

9. If  $y = \sin a\theta \cos b\theta$  then  $y' =$

- (A)  $-ab \cos a\theta \sin b\theta$       (B)  $a \cos a\theta \cos b\theta + b \sin \theta \sin b\theta$       (C)  $ab \cos a\theta \sin b\theta$   
(D)  $a \cos a\theta \cos b\theta - b \sin a\theta \sin b\theta$       (E)  $a \cos a\theta ab\theta + b \sin a\theta \sin b\theta$       [9] \_\_\_\_\_

10. The graph of  $y = 3x^3 - \frac{5x^2}{2} + 6$  is concave up if

- (A)  $x > \frac{5}{18}$       (B)  $x < \frac{5}{18}$       (C)  $x > \frac{18}{5}$       (D)  $x < \frac{18}{5}$       (E)  $x = 0$       [10] \_\_\_\_\_

**SCORING:** 20 points for each correct answer, -5 for each wrong answer.

---

1. The equation of the line normal to (perpendicular to the tangent line) the curve  $y = 2x^2 + x - 1$  at  $x = 2$  is:

- (A)  $y = 9x + 27$       (B)  $9y = -x + 83$       (C)  $9y = -x + 11$       [1] \_\_\_\_\_  
 (D)  $9y = x + 11$       (E)  $9y = x + 83$

2.  $(-\infty, -2) \cap (-\infty, 5]$  is:

- (A)  $(-\infty, 5]$       (B)  $(-\infty, -2)$       (C)  $[-3, 5]$       (D)  $[-2, 5]$       (E) none of these      [2] \_\_\_\_\_

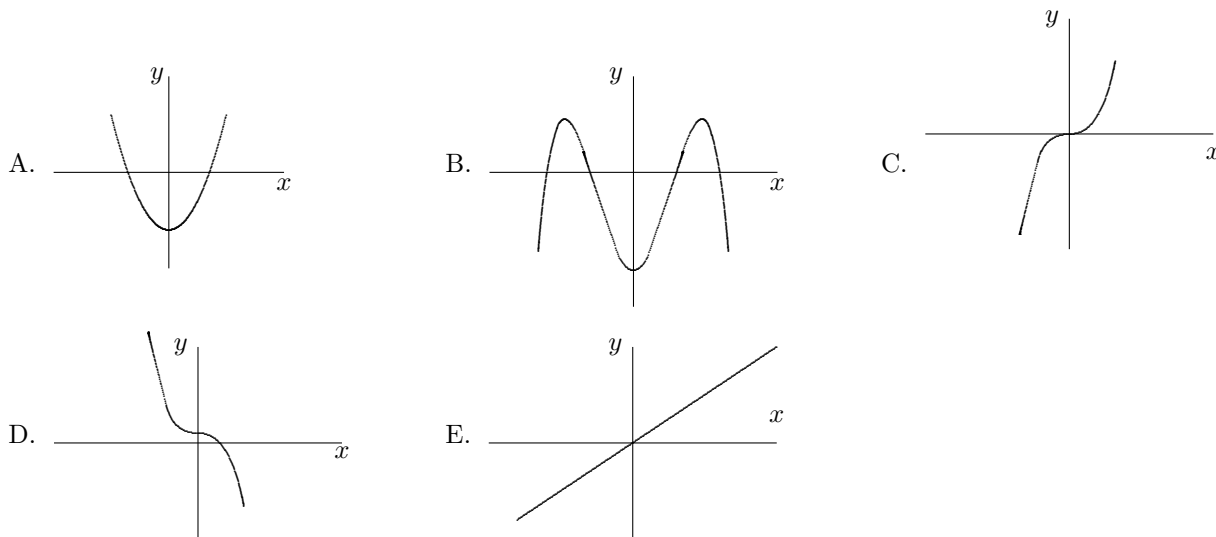
3. The number of positive roots of  $x^5 - 2x^4 - 2x^2 - x = 0$  is:

- (A) 3      (B) 1      (C) 0      (D) 2      (E) 2 or 4      [3] \_\_\_\_\_

4.  $(223)(15) = 4011$  in base  $b$ . So  $b = ?$

- (A) 9      (B) 8      (C) 7      (D) 6      (E) none of these      [4] \_\_\_\_\_

5. Which function has the second derivative negative for all  $x > 0$ ?

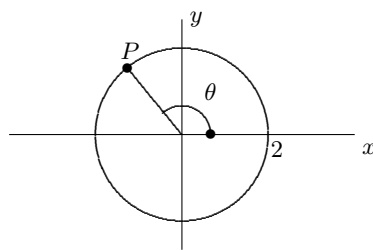


[5] \_\_\_\_\_

6.  $3 + 1 + \frac{1}{3} + \frac{1}{9} \cdots = ?$

- (A)  $\frac{3}{2}$       (B)  $\frac{9}{2}$       (C)  $\infty$       (D) 6      (E) none of these      [6] \_\_\_\_\_

7. The polar coordinates of  $p$  are:



- (A)  $(1, \sin \theta)$       (B)  $(1, \cos \theta)$       (C)  $(-2, \cos \theta)$       (D)  $(-2, \theta)$       (E)  $(2, \theta)$       [7] \_\_\_\_\_

8. The sum of the first 10 terms of the series  $-1 + \frac{1}{2} - \frac{1}{4} \cdots$  is:

- (A)  $\frac{1}{3 \cdot 1024} - \frac{2}{3}$       (B)  $\frac{1}{3 \cdot 512} - \frac{2}{3}$       (C)  $-\frac{2}{3}$       (D)  $+\frac{2}{3}$       (E)  $\frac{1}{3 \cdot 1.024} + \frac{2}{3}$       [8] \_\_\_\_\_

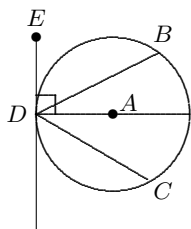
9.  $(667)^{2001}$  has last digit:

- (A) 9      (B) 6      (C) 3      (D) 2      (E) 7      [9] \_\_\_\_\_

10.  $\cos(-15^\circ)$  is exactly:

- (A)  $\frac{2 + \sqrt{3}}{4}$       (B)  $\frac{2 - \sqrt{3}}{4}$       (C)  $\frac{\sqrt{2 + \sqrt{3}}}{2}$       (D)  $\frac{\sqrt{2} - \sqrt{3}}{2}$       (E)  $-\frac{\sqrt{2 + \sqrt{3}}}{2}$       [10] \_\_\_\_\_

SCORING: 20 points for each correct answer, -5 for each wrong answer.



1. If  $A$  is the center of a circle  $BDC$ ,  $\angle BAC = 60^\circ$  and  $DE$  is  $\perp$  to  $AD$ , then  $\angle EDB$  is:

- (A)  $30^\circ$       (B)  $45^\circ$       (C)  $50^\circ$       (D)  $75^\circ$       (E)  $35^\circ$       [1] \_\_\_\_\_

2.  $y = 2 + 3 \sin 6x$  is periodic with period

- (A) 6      (B)  $6\pi$       (C)  $\frac{\pi}{3}$       (D)  $\frac{\pi}{2}$       (E)  $3\pi$       [2] \_\_\_\_\_

3.  $\lim_{x \rightarrow 0} \frac{3 \cos 2x}{x^3 - 2x + 7} =$

- (A) not defined      (B)  $\frac{1}{7}$       (C)  $\frac{3}{7}$       (D) 0      (E)  $\frac{3}{2}$       [3] \_\_\_\_\_

4. The coefficient of  $x^4$  in the product  $(x^6 + 3x^4 - x^3 + 7x^2 - x + 1)(x^5 - x^4 + x^2 - x + 3)$  is

- (A) 9      (B) 12      (C) -3      (D) 0      (E) none of these      [4] \_\_\_\_\_

5.  $e^{2x} + 5e^x + 4 = 0$  has how many real solutions?

- (A) none      (B) 1      (C) 2      (D) 3      (E) infinitely many      [5] \_\_\_\_\_

6. The product of the roots of  $3x^4 - 2x^2 + 5$  is:

- (A) 5      (B) -2      (C)  $-\frac{5}{3}$       (D)  $\frac{5}{3}$       (E)  $-\frac{2}{3}$       [6] \_\_\_\_\_

7.  $2 \sin 3A \cos 5A =$

- (A)  $\sin 8A$       (B)  $\sin 15A^2$       (C)  $\sin 15A$       (D)  $\cos 8A \cos 2A$       (E)  $\sin 8A - \sin 2A$       [7] \_\_\_\_\_

8. The equation of the line tangent to the curve  $y = e^x + 3$  at  $(0,4)$  is:

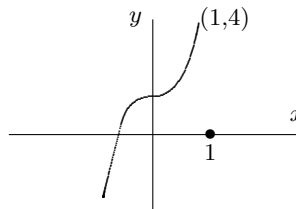
- (A)  $y = x - 4$       (B)  $y = x + 4$       (C)  $y = 4(x - 1)$       (D)  $y = 4x - 1$       (E)  $y = 4$       [8] \_\_\_\_\_

9. If  $|4 - 2x| < 6$  then

- (A)  $-1 < x < 5$       (B)  $1 < x < -5$       (C)  $-1 < x < -5$   
(D)  $x < -5$  or  $x > 1$       (E)  $x < -1$  and  $x < -5$

[9] \_\_\_\_\_

10. A region is bounded by  $y = x^3 + 3$ , the line  $y = 4$  and the  $y$ -axis. The area of the region is:



- (A)  $\int_0^4 (x^3 - 3) dx$  (B)  $\int_3^4 (y - 3) dy$  (C)  $\int_3^4 (y - 3)^{\frac{1}{3}} dy$  (D)  $\int_3^4 (y^{\frac{1}{3}} - 3) dy$  (E)  $\int_0^4 (y - 3)^{\frac{1}{3}} dy$  [10] \_\_\_\_\_

Grades HS  
2001 Math Contest Exam

Exam	T1	T2	T3	T4	T5
P1	c	c	e	b	d
P2	e	b	c	b	c
P3	b	c	d	b	c
P4	c	e	c	c	e
P5	a	c	b	d	a
P6	a	d	d	b	d
P7	d	e	b	e	e
P8	d	b	c	b	b
P9	d	c	d	e	a
P10	c	d	a	c	c