WCTM MATHEMATICS CONTEST, 2000

Test 1 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

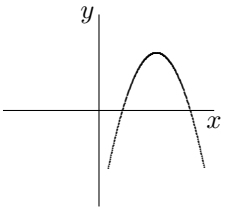
1. 1315-216=1066 in base b. So $b =$

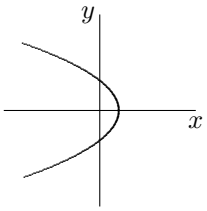
(A) 8 (B) 7 (C) 11 (D) 9 (E) 12 [1] _____

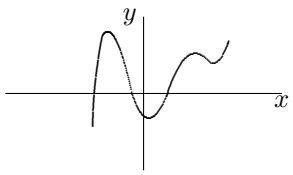
2. Acme Medical Supply Company sells supplies to a hospital on a "cost, plus 10% of the final price" price schedule. An item costs Acme $45. The hospital pays Acme:

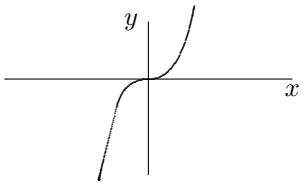
(A) $49.50 (B) $50.50 (C) $50 (D) $45.45 (E) $51 [2] _____

3. Which graphs are functions whose inverse graphs are also functions?

1. 

2. 

3. 

4. 

(A) 1 only (B) 2 only (C) 3 only (D) 4 only (E) 1 and 2 only [3] _____

4. If $|3x - 5| < 4$ then

(A) $-4 < x < 4$ (B) $x < 3$ (C) $\frac{1}{3} < x < 3$ (D) $-\frac{1}{3} < x < 3$ (E) $3 < x < 9$ [4] _____

5. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $c =$

(A) 2 (B) -2 (C) $-\frac{1}{5}$ (D) 3 (E) -3 [5] _____

6. The sum $2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} \cdots + 2\left(\frac{2^n}{3^n}\right) \cdots$ is

(A) 3 (B) 3/5 (C) 6 (D) 6/5 (E) none of these [6] _____

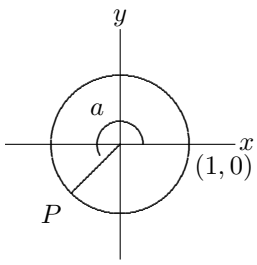
7. The graph of $y = x^4 - 3x^2$ is concave down when

(A) $x > 1$ (B) $x = 1$ (C) $x < 1$ (D) $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ (E) $x = 0$ or $x = 3$ [7] _____

8. The last digit of 1732^{2000} is:

(A) 2 (B) 6 (C) 4 (D) 8 (E) 0 [8] _____

9. The coordinates of P are:



(A) $(\cos\ a, \sin\ a)$ (B) $(-cos\ a, -\sin\ a)$ (C) $(cos\ a, -\sin\ a)$
(D) $(\sin\ a, \cos a)$ (E) none of these [9] _____

10. $-3 + 4 + 11 + \cdots + 697 =$

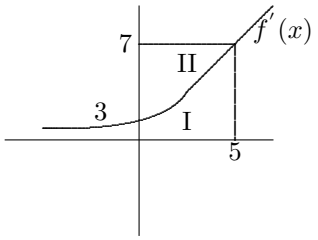
(A) 7000 (B) 69,001 (C) 35,047 (D) 70,999 (E) 35,047 [10] _____

Test 2 A/B *Name*: _____ *School*: _____

1. The length of the vector $3i + 4j + 5k$ is:

(A) $5\sqrt{2}$ (B) $\sqrt{19}$ (C) 19 (D) $\sqrt{17}$ (E) none of these [1] _____

2. For the curve $f'(x)$ in the picture, $f'(0) = 3, f'(5) = 7$. The ratio of the area I:II is:



(A) $\frac{7}{30}$ (B) $\frac{7}{23}$ (C) $\frac{4}{30}$ (D) $\frac{4}{23}$ (E) more information needed [2] _____

3. The number of subsets of size 3 that can be selected from a set of 8 distinct elements is:

(A) 24 (B) 56 (C) 336 (D) 28 (E) 256 [3] _____

4. If $x^3, x^t, \ldots$ is a geometric sequence and the 8th term is x^{52}, then $t =$

(A) 8 (B) 9 (C) 10 (D) 11 (E) $\frac{9}{2}$ [4] _____

5. $\cos(-75°)$ is exactly:

(A) $\frac{(\sqrt{2}-\sqrt{6})}{2}$ (B) $\frac{(\sqrt{2}+\sqrt{6})}{4}$ (C) $\frac{(\sqrt{-6}-\sqrt{2})}{4}$ (D) $\frac{(\sqrt{6}-\sqrt{2})}{4}$ (E) none of these [5] _____

6. Equilateral triangles are inscribed in circles of radius 1 and 2 as shown. The ratio of the area of the larger triangle to the area of the smaller triangle is:

(A) 2:1 (B) 4:1 (C) 3:2 (D) 5:4 (E) none of these [6] _____

7. A bag contains three \$5 bills and one \$10 bill. A game consists of your paying n dollars for the privilege of reaching in and drawing out a bill which you get to keep. If the game is fair, $n =$

(A) \$7.67 (B) \$8.25 (C) \$17.50 (D) \$6.25 (E) none of these [7] _____

8. Teacher: "Kwan, how long did you take to do your homework?" Kwan: I did it yesterday afternoon between 4:00 and 5:00. I remember the hands were together when I started and exactly opposite when I finished." To the nearest minute, Kwan spent how much time on the homework?

(A) 30 minutes (B) 35 minutes (C) 32 minutes (D) 33 minutes (E) 34 minutes [8] _____

9. A committee of 4 men and 5 women selected from a group of 14 men and 20 women. The number of different ways that this can be done is:

(A) $\binom{34}{9}$ (B) $\binom{14}{4}\binom{20}{5}$ (C) $\binom{14}{4}+\binom{20}{5}$

(D) $\dfrac{\binom{14}{4}\binom{20}{5}}{\binom{34}{9}}$ (E) none of these [9] _____

10. If $[3(230+x)]^2 = 492,t04$, then $t =$

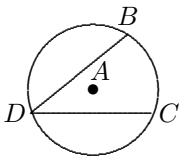
(A) 8 (B) 7 (C) 6 (D) 5 (E) 4 [10] _____

Test 3 A/B *Name*: _____ *School*: _____

1. P and Q have coordinates $(1, a)$ and $(3, 2a)$. The length of segment PQ is:

(A) $2 + a$ (B) $2 = \dfrac{3a}{2}$ (C) $\sqrt{a^2 + 4}$ (D) $\sqrt{a + 2}$ (E) $a^2 + 4$ [1] _____

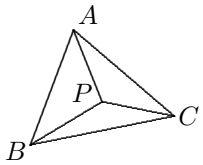
2. In the circle with center A, points B, C, D are on the circle as shown. If $\angle BAC$ is 60 degrees, then $\angle BDC$ is



(A) 45 degrees (B) 30 degrees (C) 60 degrees

(D) depends on the exact location of D (E) none of these

[2] _____

3. 

If line PA bisects $\angle A$ and line PB bisects $\angle B$, then line PC:

(A) never bisects $\angle C$ (B) bisects $\angle C$ only if the triangle is equilateral

(C) always bisects $\angle C$ (D) bisects $\angle C$ only if line PC is the perpendicular bisector of side AB [3] _____

(E) none of these

4. The table contains all positive integers:

A	B	C	D	E	F	G
			1	2	3	4
5	6	7	8	9	·	·
·	·	·	·	·	·	·

. 2000 is located in:

(A) row 285 and column A (B) row 287 and column A (C) row 285 and column B

(D) row 287 and column B (E) row 287 and column C

[4] _____

5. If $y = 2\cos\dfrac{\pi x}{2}$, then y is periodic with period p and amplitude a. So

(A) $p = 4,\ a = 4$ (B) $p = 4\pi,\ a = 2$ (C) $p = 4,\ a = 2$ (D) $p = 4,\ a = -2$ (E) $p = 2,\ a = 2$ [5] _____

6. If $b^y = a$ and $b > 0$, then

(A) $b = \log_a y$ (B) $y = \log_b a$ (C) $b = \log_y a$ (D) $y = \log_a b$ (E) $a = \log_e y$ [6] _____

7. $\lim_{x \to 0} \dfrac{\left(3x^2 - 5x + 4\right)}{(12x^2 + 17x)} =$

(A) 0 (B) $\dfrac{1}{4}$ (C) $-\dfrac{5}{17}$ (D) does not exist (E) none of these [7] _____

8. The coefficient of x^3 in the expansion of $\left(x^5 + 3x^4 + 5x^2 - 2\right)\left(x^7 - 2x^6 + 4x^3 - 5x^2 + 2x - 1\right)$ is

(A) 27 (B) 2 (C) 19 (D) 21 (E) -8 [8] _____

9. Let N be the smallest positive integer such that if N is divided by 5 the remainder is 3 and if N is divided by 9 the remainder is 7 and if N is divided by 7 the remainder is 4. Then:

(A) $0 < N < 21$ (B) $20 < N < 40$ (C) $40 < N < 61$ (D) $60 < N < 81$ (E) $80 < N < 100$ [9] _____

10. In $\triangle ABC$, $\angle B$ is 120 degrees, side $a = 5$ and side $b = 8$. Then tan $A =$ (two-digit accuracy)

(A) .54 (B) .57 (C) 2.2 (D) .64 (E) .33 [10] _____

Test 4 A/B *Name*: _____ *School*: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. The year 2000 ends in 000 in base 10 notation. But in base 8 notation the year 2000 is 3720. The first positive integer which ends in 000 in both base 8 notation and base 10 notation is: (In base 10 notation)

   (A) less than 19,999 (B) between 19,999 and 39,999 (C) between 39,999 and 59,999
   (D) between 59,999 and 79,999 (E) none of these

   [1] _____

2. Jan and Beth work in a factory counting widgets. Jan, in 5 hours, can count a number n which Beth can count in eleven hours. How large is the number both can count in 7 hours?

   (A) $2n$ (B) $8n$ (C) $6n$ (D) $16n$ (E) $\frac{112n}{55}$ [2] _____

3. The solution $2^x + 2^{x-1} = 10$ is exactly:

   (A) $\log \frac{20}{3} - \log 2$ (B) $\frac{\log 20}{\log 3}$ (C) $\log 20 - \log 3$ (D) $\log \frac{20}{6}$ (E) $\frac{\log 20 - \log 3}{\log 2}$ [3] _____

4. The distance from line $4y + 3x = 25$ to the point $(0, 0)$ is:

   (A) 3 (B) 4 (C) 5 (D) $\sqrt{5}$ (E) $\sqrt{3}$ [4] _____

5. If $\log x - \log y = 3$ and $x - y = 666$, then $x + y =$

   (A) $\frac{2002}{3}$ (B) $\frac{1998}{3}$ (C) $\frac{2000}{3}$ (D) $\frac{1000}{999}$ (E) none of these [5] _____

6. The sum of the roots $x^3 - 2x^2 + 5$ is:

   (A) 5 (B) 0 (C) $-\frac{5}{2}$ (D) 2 (E) -2 [6] _____

7. If $\frac{x}{y} = \frac{3}{4}$, which of the following is WRONG?

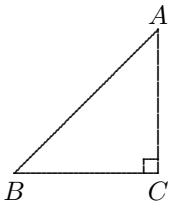
   (A) $\frac{x+y}{x} = \frac{7}{3}$ (B) $\frac{y}{y-x} = 4$ (C) $\frac{x-2}{x} = \frac{-5}{3}$ (D) $\frac{x}{2y} = \frac{3}{8}$ (E) $\frac{x}{2y} = -\frac{3}{8}$ [7] _____

8. A positive number is added to $\frac{1}{2}$ of its reciprocal. The smallest such sum is:

(A) $\sqrt{3}$ (B) 3 (C) $\sqrt{2}$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{2}}{2}$ [8] _____

9. If x is an angle in standard position, then $\frac{2\sin\ x}{\sin\ 2x} =$

(A) 1 (B) $-\cos\ x$ (C) $\frac{2\sin\ x}{1-\cos^2\ x}$ (D) $\cos\ x$ (E) $\sec\ x$ [9] _____

10. 

If $\tan\ A > \tan\ B$ in the triangle, then

(A) $\sin A < \cos A$ (B) $\cos A < \sin A$ (C) $\sin A < \sin B$ (D) $\cos B < \sin A$ (E) $\cos A < \sin B$ [10] _____

Test 5 A/B *Name*: _____ *School*: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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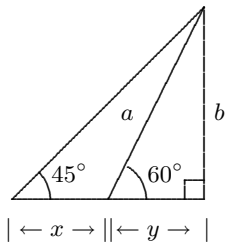
1. The augmented matrix for a system of linear equations reduces to $\begin{pmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. The solution set is:

(A) $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$ (C) $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + k \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$ (D) $\begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$ (E) the empty set [1] _____

2. 

In the diagram

(A) $a = 2x$ (B) $a = b + x$ (C) $a = 2(b - x)$ (D) $a^2 = b^2 + x^2$ (E) none of these [2] _____

3. If $F'(x) = \dfrac{3}{x^2}$ and $F(2) = 4$ then $F(x) =$

(A) $-\dfrac{3}{x}$ (B) $\dfrac{3}{x}$ (C) $-\dfrac{3}{x} + 1\frac{1}{2}$ (D) $-\dfrac{6}{x^{-3}} + \dfrac{5}{2}$ (E) none of these [3] _____

4. If $e^{2x} - 3e^x + 2 = 0$, then:

(A) there are 2 unequal real roots (B) there is one root with multiplicity (C) there is exactly one root
(D) there are two unequal roots that are not real (E) none of these

[4] _____

5. The equation of the line tangent to the curve $y = 2x^2 + x - 1$ at $x = 2$ is:

(A) $y = 9x + 1$ (B) $y = 9x - 9$ (C) $y = 2x - 1$ (D) $y = 2x + 1$ (E) $y = \dfrac{x}{9} - 19$ [5] _____

6. $(-\infty, -2) \cap [-3, \infty) =$

(A) $(-\infty, \infty)$ (B) $(-2, -3]$ (C) $[-3, -2)$ (D) $(-\infty, -3]$ (E) $[-3, \infty]$ [6] _____

7. If $x^2 + 6x = -5$, then $x + 4 =$

(A) -1 or 3 (B) 1 or -3 (C) -1 or -3 (D) -1 or 7 (E) 1 or -7 [7] _____

8. Let $\overline{z}$ be the complex conjugate of z. Which of the following is sometimes FALSE?

(A) $\overline{z+w} = \overline{z} + \overline{w}$ (B) $\overline{(z/w)} = (\overline{z})/(\overline{w})$ (C) $\overline{z}^4 = \overline{z^4}$ (D) $\overline{\overline{z}} = z$ (E) $\overline{z} = \dfrac{1}{(\overline{z})}$ [8] _____

9. The number of nonnegative roots of $x^5 + 2x^4 - 2x^2 - x = 0$ is:

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 [9] _____

10. The difference between two numbers a and b is 21. If the larger number is divided by the smaller the quotient is 4 with remainder 3. The sum of a and b is:

(A) 30 (B) 12 (C) 33 (D) 17 (E) 21 [10] _____

Grades HS
2000 Math Contest Exam

Exam	T1	T2	T3	T4	T5
P1	b	a	c	d	c
P2	c	e	b	e	c
P3	d	b	c	e	e
P4	c	c	b	c	a
P5	d	d	c	a	b
P6	c	b	b	d	c
P7	d	d	d	e	a
P8	b	d	b	c	e
P9	a	b	e	e	b
P10	c	a	d	b	c