

WCTM MATHEMATICS CONTEST, 1998

Test 1

NAME: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The equation $\frac{2}{x-2} + \frac{x-1}{2-x} = \frac{4-x}{x-2}$ is satisfied by:

- (A) only $x = 2$
- (B) exactly two values of x
- (C) no values of x
- (D) infinitely many values of x , including $x = 2$
- (E) all values of x except $x = 2$

[1] _____

2. $|y + 3| > 4$ is true if and only if

- (A) $y > 1$
- (B) $y < -7$
- (C) $1 < y < -7$
- (D) $1 > y > -7$
- (E) none of these

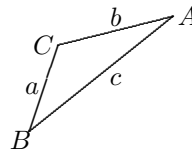
[2] _____

3. Figure $ABCD$ is a trapezoid (you sketch it) but not a parallelogram. So:

- (A) no two sides can be equal
- (B) no two angles can be equal
- (C) the diagonals bisect each other
- (D) the nonparallel sides are always equal
- (E) none of these

[3] _____

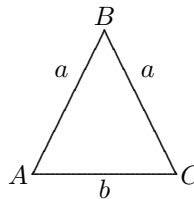
4. If $\angle C > 90^\circ$ and $\triangle ABC$ has area k , then:



- (A) $\frac{a}{\sin A} = \frac{b}{\sin B}$
- (B) $\sin A = \frac{k}{bc}$
- (C) $\sin A = \frac{2k}{ac}$
- (D) $\sin A = \frac{2k}{ab}$
- (E) $\sin A = \frac{k}{ac}$

[4] _____

5. Isosceles triangle ABC has sides a, a, b . The perimeter of the triangle formed by joining the midpoints of the sides of $\triangle ABC$ is:



- (A) $\frac{1}{2}(a + b)$
- (B) $a + \frac{b}{2}$
- (C) $\frac{1}{2}a + b$
- (D) $2a + b$
- (E) $\frac{2a}{3} + \frac{b}{2}$

[5] _____

6. In base b the computation $(11_b)(1221_b) = 13431_b$ is correct for:

- (A) $b = 4$ only (B) $b = 4$ and $b = 5$ (C) any integer $b, b > 4$
(D) any even integer b (E) none of these

[6] _____

7. How many different arrangements of the letters "WYOMING COWBOYS" are there?

- (A) $14!$ (B) $\frac{14!}{2!3!}$ (C) $\frac{26!}{10!16!}$ (D) $\frac{14!}{2!2!3!}$ (E) none of these

[7] _____

8. Jo drove half way home averaging 15mph. How fast must she drive the rest of the way in order to average 30mph for the entire trip?

- (A) 60mph (B) 45mph (C) 40mph (D) there is no solution to this problem (E) there is no solution but it is not listed

[8] _____

9. $\log \sin 45^\circ =$

- (A) $\frac{\log^2}{2}$ (B) $\log\left(\frac{2}{2}\right)$ (C) $-\frac{\log 2}{2}$ (D) $\log \frac{1}{2}$ (E) none of these

[9] _____

10. If $\tan A = \cot A$ and $90^\circ \leq A \leq 180^\circ$ then:

- (A) $A = 45^\circ$ (B) $A = 90^\circ$ (C) $A = 120^\circ$ (D) $A = 135^\circ$ (E) $A = 145^\circ$

[10] _____

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Test 2

NAME: _____

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SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. $\frac{a-b}{c-d}$ is equal to:

- (A) $\frac{b-c}{d-a}$ (B) $\frac{a+b}{c+d}$ (C) $\frac{b-a}{d-c}$ (D) $\frac{a-b}{d-c}$ (E) none of these [1] _____

2. $\frac{3(10)^{-5} \times 4(10)^7 \times 9 \cdot (10)^0}{27(10)^1(.003987)}$ is closest to:

- (A) 10^4 (B) $4 \cdot 10^3$ (C) .06398 (D) 16 (E) .16398 [2] _____

3. How much of a 5% acid solution must be mixed with a 20% acid solution to produce 2 gallons of a 15% solution?

- (A) 2/3 gallon (B) 4/3 gallon (C) 3/4 gallon (D) 1 gallon (E) none of these [3] _____

4. If $a = 3$, $b = 5$, and $c = -4$ then $c \left[a - b(c - a) + \frac{b}{a + c} \right] =$

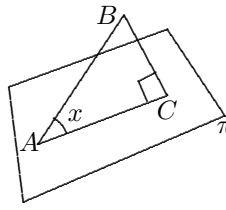
- (A) 132 (B) 28 (C) -36 (D) -132 (E) none of these [4] _____

5. The length of a non-face diagonal of a $2 \times 2 \times 2$ cube is:

- (A) 2 (B) $\sqrt{5}$ (C) $2\sqrt{3}$ (D) $\sqrt{6}$ (E) 6 [5] _____

6. The maximum value of $y = 3x^2 - 6x + 1$ on the interval $[-1, 4]$ is:

- (A) 1 (B) 10 (C) 25 (D) 49 (E) none of these [6] _____



7. Points A, C are in plane π , B is not. $AC \perp BC$ and $\angle A = x$. Then:

- (A) $\overline{AC} = \overline{AB} \sin x$ (B) $\overline{AC} = \overline{AB} \cos x$ (C) $\overline{AC} = \overline{AB} \tan x$
 (D) $\overline{AC} = \overline{AB} \cot x$ (E) none of these

[7] _____

8. If $\log_b x = .731$ then $\log_b \left(\frac{x^2}{2} \right) =$

- (A) $\frac{2(.731)^b}{2}$ (B) $2(.731) - \log_b 2$ (C) $2 \log(.731) - \frac{1}{2}$ (D) $2(.731) - 2$ (E) $\frac{.731}{2}$

[8] _____

9. The area in the first quadrant bounded by $y = \sqrt{4 - x^2}$ and the x and y -axes is:

- (A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) $\int_0^4 \sqrt{1 - x^2} dx$ (E) none of these

[9] _____

10. The coefficient of x^{10} in the expansion of $(x^2 - 5)^7$ is:

- (A) -525 (B) $\frac{7!}{10!3!}$ (C) 525 (D) 25 (E) none of these

[10] _____

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Test 3

NAME: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The graph of $y = x^2 + 4x + k$ crosses the x -axis at exactly two distinct points for:

- (A) all $k > 2$ (B) all $k > 4$ (C) all $k < 2$ (D) all $k < 4$ (E) the graph never crosses the axis [1] _____

2. An equation of the line tangent to $y = 3x^2 - 5x + 2$ at $(0, 2)$ is:

- (A) $y = -5x - 2$ (B) $y = 7x - 2$ (C) $y = -5x + 2$ (D) $y = 7x + 2$ (E) $y = -7x + 2$ [2] _____

3. The last two digits of $(1998)^{100}$ are:

- (A) 24 (B) 96 (C) 76 (D) 48 (E) none of these [3] _____

4. If $x = \frac{(v_1 + v_2)h}{v_3}$ then

- (A) $h = \frac{v_3x - v_1}{v_2}$ (B) $h = \frac{v_3x}{v_1 + v_2}$ (C) $h = \frac{v_3x}{v_1v_2}$ (D) $h = \frac{v_3x - v_1x}{v_2}$ (E) none of these [4] _____

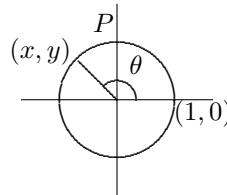
5. The sum of $(-1) + (3) + (7) + (11) + \dots + 1999$ is:

- (A) 1,000,998 (B) 999,000 (C) 500,000 (D) 500,499 (E) none of these [5] _____

6. If $\frac{4x + 2}{x^2 + x - 6} = \frac{A}{x + 3} + \frac{B}{x - 2}$, then

- (A) $AB = 4$ (B) $AB = -4$ (C) $AB = 0$ (D) $AB = 1$ (E) none of these [6] _____

7. The coordinates (x, y) of point P are:



- (A) $(\sin \theta, \cos \theta)$ (B) $(-\sin \theta, \cos \theta)$ (C) $(-\cos \theta, \sin \theta)$ (D) $(\cos \theta, -\sin \theta)$ (E) none of these [7] _____

8. For any Rhombus with acute angle θ , the ratio of the long diagonal to the short diagonal is:

- (A) $\tan \theta$ (B) $\tan \left(\frac{\theta}{2} \right)$ (C) $\frac{\tan \theta}{2}$ (D) $\cot \frac{\theta}{2}$ (E) $\cot \theta$ [8] _____

9. $y = 3 \sin \frac{\pi}{2}x$ is a periodic function with period:

- (A) π (B) 2 (C) 2π (D) 1 (E) 4 [9] _____

10. Josley's Department Store has a 40% discount on the price of all items and a particular item has an additional 10% off the sale price. The final discount is what percent of the original price of the item?

- (A) 50% (B) 45% (C) 46% (D) 44% (E) none of these [10] _____

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Test 4

NAME: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. An equation of the circle with center $(2, -3)$ and diameter 4 is:

- (A) $x^2 + y^2 - 4x + 6y + 9 = 0$
- (B) $x^2 + y^2 + 4x - 6y = 0$
- (C) $x^2 + y^2 + 4x - 6y + 13 = 0$
- (D) $x^2 + y^2 - 4x - 6y + 9 = 0$
- (E) $x^2 + y^2 - 4x - 6y + 13 = 0$

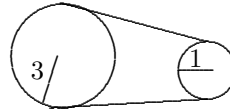
[1] _____

2. The sum and product of the roots of $x^4 - 4x^3 + x^2 + 6x$ are respectively:

- (A) $-4, 0$
- (B) $4, 0$
- (C) $-4, 6$
- (D) $4, -6$
- (E) none of these

[2] _____

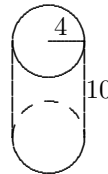
3. A pulley system (as shown) has the large pulley making 10 revolutions per minute (rpm). The small pulley is making how many rmp?



- (A) 30 rpm
- (B) 60 rpm
- (C) $3 \frac{1}{3}$ rpm
- (D) $7 \frac{1}{2}$ rpm
- (E) none of these

[3] _____

4. A cylindrical tank of radius 4' and height 10' can be filled with fertilizer for \$100. A spherical tank with radius 4 is filled with fertilizer at the same unit price. How much does it cost to fill the spherical tank (to the nearest cent)?



- (A) \$50
- (B) \$147.33
- (C) \$40.33
- (D) \$53.33
- (E) none of these

[4] _____

5. $\lim_{n \rightarrow \infty} \frac{-n^2 - n + 17}{3n^2 + 4n - 73} =$

- (A) ∞
- (B) $-\infty$
- (C) 0
- (D) 1
- (E) $-\frac{1}{3}$

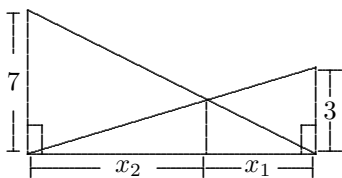
[5] _____

6. If $(x + 1)(x - 3) = 6$ then the sum of the roots is:

- (A) -2
- (B) 9
- (C) 2
- (D) 4
- (E) $\sqrt{-32}$

[6] _____

7. The ratio $\frac{x_1}{x_1 + x_2}$ is:



- (A) $\frac{3}{7}$ (B) $\frac{3}{4}$ (C) $\frac{7}{10}$ (D) $\frac{4}{7}$ (E) $\frac{3}{10}$ [7] _____

8. An equation of the perpendicular bisector of the segment \overline{PQ} where $P = (4, -3)$ and $Q = (-2, 1)$ is:

- (A) $y = 3x - 1$ (B) $2y = 3x - 5$ (C) $y = -3x - 5$ (D) $y = -\frac{1}{3}x - 1$ (E) none of these [8] _____

9. $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$ where $f(x) = x^7 - 3x^6$ is:

- (A) -128 (B) 128 (C) -64 (D) 64 (E) none of these [9] _____

10. If $\sin A = -\frac{3}{5}$ and $\cos A = -\frac{4}{5}$ then $\cos 2A =$

- (A) $-\frac{7}{25}$ (B) $\frac{18}{25}$ (C) $\frac{7}{25}$ (D) $\frac{24}{25}$ (E) $-\frac{24}{25}$ [10] _____

WCTM MATHEMATICS CONTEST, 1998

Test 5

NAME: _____

CLASS AB

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. If z is a non-zero complex number and \bar{z} is the conjugate of z and $|z|$ is the length of z , then:

- (A) $\frac{1}{z} = \frac{1}{|z|}$ (B) $\frac{1}{z} = \bar{z}$ (C) $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$ (D) $\frac{1}{z} = |z|\bar{z}$ (E) none of these [1] _____

2. If $\left[\begin{array}{cc|c} 1 & 3 & -6 \\ 2 & -1 & 2 \end{array} \right]$ is the augmented matrix for a system of linear equations with variables x_1, x_2 then

- (A) $x_1 = 0, x_2 = -2$ (B) $x_1 = 2, x_2 = 0$ (C) $x_1 = 0, x_2 = 2$ [2] _____
 (D) $x_1 = -2, x_2 = 0$ (E) $x_1 = -6, x_2 = -2$

3. When $(x^5 - 3x^4 + 7x^3 - 2x^2 + 1)(x^7 + 3x^6 - 4x^3 + 7x^2 - x + 2)$ is written as a polynomial, the coefficient of x^7 is

- (A) -9 (B) 8 (C) 0 (D) 6 (E) 20 [3] _____

4. If $4\sin^2\theta + 2\cos^2\theta = 3$ and $0^\circ \leq \theta \leq 360^\circ$, then $\theta =$

- (A) 45° or 135° (B) 45° (C) $45^\circ, 135^\circ, 225^\circ$ or 315° (D) 45° or 315° (E) 45° or 225° [4] _____

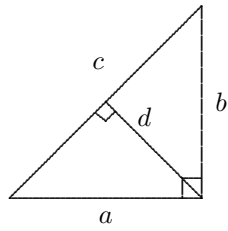
5. Let $P = (-3, 4)$ and $Q = (5, -8)$ be points in the plane. If R is on \overline{PQ} with $\frac{|PQ|}{|PR|} = 4$, then the coordinates of R are:

- (A) $(-1, 1)$ (B) $(-1, 7)$ (C) $(3, -5)$ (D) $(3, 5)$ (E) none of these [5] _____

6. How many 2-digit odd numbers can be formed by choosing two distinct digits from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

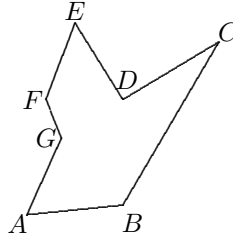
- (A) 16 (B) 20 (C) 15 (D) 40 (E) none of these [6] _____

7. In the figure, $d =$



- (A) $\frac{ac}{b}$ (B) $\frac{bc}{a}$ (C) $\frac{ab}{c}$ (D) $\frac{c}{ab}$ (E) $\frac{a}{bc}$ [7] _____

8. The sum of the indicated angles A, B, C, D, E, F, G is:



- (A) 540° (B) 900° (C) 360° (D) 1080° (E) 720° [8] _____

9. If $S = \frac{1}{3} - \frac{1}{12} + \frac{1}{48} - \frac{1}{192} \dots$ then $S =$

- (A) $\frac{4}{5}$ (B) $\frac{4}{15}$ (C) $\frac{4}{9}$ (D) $\frac{5}{4}$ (E) none of these [9] _____

10. n is a positive whole number. When n is divided by 3 the remainder is 1. When n is divided by 5 the remainder is 2 and when n is divided by 4 the remainder is 3. The smallest possible value for n is:

- (A) 67 (B) 127 (C) 22 (D) 37 (E) none of these [10] _____

Grades HS
1998 Math Contest Exam

Exam	T1	T2	T3	T4	T5
P1	C	C	A	D	C
P2	E	A	B	C	A
P3	E	A	A	C	E
P4	A	D	D	B	C
P5	B	C	E	D	A
P6	C	C	C	A	D
P7	D	B	E	E	C
P8	D	B	B	D	B
P9	C	B	A	E	B
P10	D	C	C	C	E