WCTM MATHEMATICS CONTEST, 1995

Test I A/B **NAME**: _____

CLASS **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. Consider the graph of $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and x and y are real variables. In how many points do the two graphs intersect?

   (A) exactly 4 (B) exactly 2 (C) at least 1, but there may be more for different values of A

   (D) 0 for at least one value of A (E) none of these

   [1] _____

2. A wooden cube with edge length n units (n an integer greater than 2) is painted black all over. The cube is cut into n^3 unit cubes by slices parallel to its faces. If the number of unit cubes with exactly one face painted black is equal to the number of unit cubes with no paint on them then n is:

   (A) 5 (B) 6 (C) 7 (D) 8 (E) none of these

   [2] _____

3. How many integers x satisfy the equation

$$(x^2 - x - 1)^{x+2} = 1?$$

   (A) 2 (B) 3 (C) 4 (D) 5 (E) none of these

   [3] _____

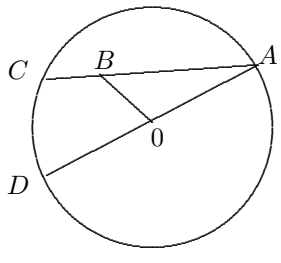
4. If $x = \dfrac{-1 + i\sqrt{3}}{2}$ and $y = \dfrac{-1 - i\sqrt{3}}{2}$, where $i^2 = -1$, then which of the following is not correct?

   (A) $x^5 + y^5 = -1$ (B) $x^7 + y^7 = -1$ (C) $x^9 + y^9 = -1$

   (D) $x^{11} + y^{11} = -1$ (E) $x^{13} + y^{13} = -1$

   [4] _____

5. In a circle with center 0, AD is a diameter, ABC is a chord, $BO = 5$ and $\angle ABO = \angle CDO = 60°$. Then the length of BC is



   (A) 3 (B) $3 + \sqrt{3}$

   (C) $5 - \dfrac{\sqrt{3}}{2}$ (D) 5

   (E) none of these

   [5] _____

6. $x(x-3) < x$ is equivalent to:

(A) $0 < x < 4$ (B) $\frac{1}{3} < x \leq 3$ (C) $x < 3$ (D) $x > 0$ (E) none of these [6] _____

7. A non-zero digit is chosen in such a way that the probability of choosing digit d is $\log_{10}(d+1) - \log_{10} d$. The probability that the digit 2 is chosen is exactly 1/2 the probability that the digit chosen is in the set

(A) $\{2, 3\}$ (B) $\{3, 4\}$ (C) $\{4, 5, 6, 7, 8\}$ (D) $\{5, 6, 7, 8, 9\}$ (E) $\{4, 5, 6, 7, 8, 9\}$ [7] _____

8. The volume of a certain rectangular solid is 8 cm^3, its total surface area is 32 cm^2, and its three dimensions are in geometric progression. The sum of the lengths of all the edges of this solid is:

(A) 28 cm (B) 32 cm (C) 36 cm (D) 40 cm (E) 44 cm [8] _____

9. Find the least positive integer n for which $\frac{n-13}{5n+6}$ is a non-zero reducible fraction.

(A) 45 (B) 68 (C) 155 (D) 226 (E) none of these [9] _____

10. The value of $\log_{500} 1000$ is?

(A) 0.5 (B) 2 (C) 3 (D) 6.908 (E) none of these [10] _____

WCTM MATHEMATICS CONTEST, 1995

Test 2 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. One student in a class of boys and girls is to be chosen to represent the class. Each student is equally likely to be chosen and the probability that a boy is chosen is 2/3 of the probability that a girl is chosen. The ratio of the number of boys to the total number of boys and girls is

   (A) 1/3 (B) 2/5 (C) 1/2 (D) 3/5 (E) 2/3 [1] _____

2. In some computer languages (such as APL), when there are no parentheses in an algebraic expression, the operations are grouped from right to left. Thus, $a \times b - c$ in such languages means the same as $a(b-c)$ in ordinary algebraic notation. If $a \div b - c + d$ is evaluated in such a language, the result in ordinary algebraic notation would be

   (A) $\frac{a}{b} - c + d$ (B) $\frac{a}{b} - c - d$ (C) $\frac{d+c-b}{a}$ (D) $\frac{a}{b-c+d}$ (E) $\frac{a}{b-c-d}$ [2] _____

3. An arbitrary circle can intersect the graph of $y = \sin x$ in

   (A) at most 2 points (B) at most 4 points (C) at most 6 points
   (D) at most 8 points (E) more than 16 points [3] _____

4. How many distinguishable rearrangements of the letters CONTEST have both the vowels first? (For instance, OETCNST is one such arrangement, but OTETSNC is not.)

   (A) 60 (B) 120 (C) 240 (D) 720 (E) 2520 [4] _____

5. Let a, a', b, b' be real numbers with a and a' nonzero. The solution to $ax+b=0$ is less than the solution to $a'x+b'=0$ if and only if

   (A) $a'b < ab'$ (B) $ab' < a'b$ (C) $ab < a'b'$
   (D) $\frac{b}{a} < \frac{b'}{a'}$ (E) $\frac{b'}{a'} < \frac{b}{a}$ [5] _____

6. The odd positive integers are arranged in 5 columns continuing the pattern shown at the right. The number 1995 appears in which column?

1	2	3	4	5
	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	33	35	37	39
47	45	43	41	
	49	51	53	55
.	.	.	.	.
.	.	.	.	.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 [6] _____

7. Let p, q and r be distinct odd prime numbers, where 1 is not considered a prime. Which of the following is the smallest positive perfect cube having $n = pq^2r^4$ as a divisor?

(A) $p^8q^8r^8$ (B) $(pq^2r^2)^3$ (C) $(p^2q^2r^2)^3$ (D) $(pqr^2)^3$ (E) $4p^3q^3r^3$ [7] _____

8. If a and b are positive numbers such that $a^b = b^a$ and $b = 9a$, then the value of a is

(A) 9 (B) $\frac{1}{9}$ (C) $\sqrt[9]{9}$ (D) $\sqrt[3]{9}$ (E) $\sqrt[4]{3}$ [8] _____

9. Six bags of marbles contain 18,19,21,23,25, and 34 marbles, respectively. One bag contains chipped marbles only. The other 5 bags contain no chipped marbles. Jane takes three of the bags and George takes two of the others. Only the bag of chipped marbles remains. If Jane gets twice as many marbles as George, how many chipped marbles are there?

(A) 18 (B) 19 (C) 21 (D) 23 (E) 25 [9] _____

10. What is the sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{2^n}$?

(A) $-\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) none of these [10] _____

WCTM MATHEMATICS CONTEST, 1995

Test 3 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. Consider a sequence $x_1, x_2, x_3, \ldots$, defined by:

$$x_1 = \sqrt[3]{3}, \quad x_2 = \left(\sqrt[3]{3}\right)^{\sqrt[3]{3}}, \quad \text{and in general,}$$

$$x_n = (x_{n-1})^{\sqrt[3]{3}} \quad \text{for } n > 1$$

What is the smallest value of n for which x_n is an integer?

(A) 2 (B) 3 (C) 4 (D) 9 (E) 27 [1] _____

2. Find the constants A and B. $\dfrac{x}{(x+1)(x+2)} = \dfrac{A}{x+1} + \dfrac{B}{x+2}$

(A) $A = 1, B = 2$ (B) $A = -1, B = 2$ (C) $A = 2, B = 2$ (D) $A = 1, B = 1$ (E) none of these [2] _____

3. Solve $x + \sqrt{5x+6} = 0$ and identify the true statement below.

(A) There are two solutions whose product is 6.
(B) There are two solutions whose product is -6.
(C) There are no solutions to this equation.
(D) There is only one solution and it is located between $-\dfrac{5}{2}$ and $-\dfrac{1}{2}$.
(E) none of the above

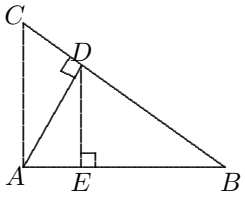
[3] _____

4. The solution of $\dfrac{x^2+3x-4}{x+3} \leq 0$ is given by

(A) $\{x| -4 \leq x \leq 1\}$ (B) $\{1\}$ (C) $\{x| -3 < x \leq 1\} \bigcup \{x|x \leq -4\}$
(D) $\{x|x \leq -4\} \bigcup \{x|x > -3\}$ (E) none of these

[4] _____

5. In the figure, length $(AC) = 75$, length $(AB) = 100$, and length $(BC) = 125$. What is the length of AE?



(A) 60 (B) 25 (C) 36 (D) 48 (E) none of these [5] _____

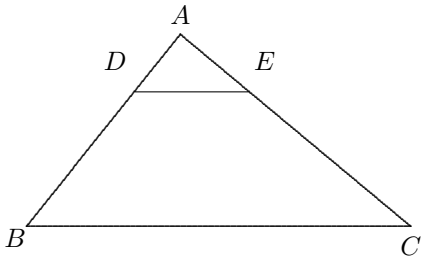
6. P, Q and R represent statements. To which of the following is the implication **if** P **then if** Q **then** R equivalent?

(A) If P and Q, then R (B) If P then (not Q or R).

(C) Both statements (A) and (B). (D) Neither statement (A) nor (B).

[6] _____

7. In the figure, AB is three times the length of AD and DE is parallel to BC. What is the ratio of the areas of triangles ABC and ADE?



(A) 4:1 (B) 2:1 (C) 3:1

(D) 1:1 (E) none of these

[7] _____

8. The equation of the line through $(-1, 3)$ perpendicular to the line $2x - 3y = 7$ is:

(A) $2x - 3y = -13$ (B) $3x + 2y = 7$ (C) $x + y = 2$

(D) $3x + 2y = 3$ (E) none of these

[8] _____

9. Which of the following are **not** lengths of the sides of any triangle?

(A) 1,3,5 (B) 3,4,5 (C) 1,3,3 (D) 2,3,4 (E) none of these [9] _____

10. A building casts a shadow of 96ft. at the same time a 6ft. tall person casts an 8ft. shadow. How tall is the building?

(A) 128 ft. (B) 94 ft. (C) 48 ft. (D) 72 ft. (E) none of these [10] _____

WCTM MATHEMATICS CONTEST, 1995

Test 4 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. The value of $\sin(\arctan(a/b))$, where $a > 0, b > 0$ is:

   (A) b/a (B) $\dfrac{a}{\sqrt{a^2+b^2}}$ (C) $\dfrac{\sqrt{3}}{2}$ (D) $\dfrac{a\sqrt{3}}{2b}$ (E) none of these [1] _____

2. What is the solution of the equation $\begin{vmatrix} 1-x & 2 \\ 3 & 2-x \end{vmatrix} = 0$?

   (A) $\{1,2\}$ (B) $\{2,3\}$ (C) $\{0\}$ (D) $\{-1,4\}$ (E) none of these [2] _____

3. $\log\left(\dfrac{x\sqrt{x-1}}{x^2+1}\right) =?$

   (A) $\dfrac{\log(x)\log\sqrt{x-1}}{\log(x^2+1)}$ (B) $10^{\frac{x\sqrt{x-1}}{x^2+1}}$ (C) $\log(x)+\dfrac{1}{2}\log(x-1)-\log(x^2+1)$

   (D) $\log(x)+\dfrac{1}{2}\log(x-1)-2\log(x+1)$ (E) none of these

   [3] _____

4. The graph of the equation $x^2-2x+y-3=0$ is:

   (A) a parabola (B) an ellipse (C) a straight line

   (D) a hyperbola (E) none of these [4] _____

5. The Supreme Court has nine members. In how many ways can a five-to-four (positive or negative)decision be reached?

   (A) 5 (B) 9 (C) 252 (D) 9! (E) none of these [5] _____

6. A company produces 105 products. Of these 63 require steel, 39 require aluminum, and 72 require copper. In addition, 44 require both steel and copper, 25 both aluminum and copper, and 20 both steel and aluminum. Finally, 13 require all three materials. How many products require at least one of these materials?

   (A) 33 (B) 66 (C) 97 (D) 98 (E) 101 [6] _____

7. Find all solutions for x that make the following two matrices equal.

$$\begin{bmatrix} 3 & 4 \\ 2 & -2x \end{bmatrix} \qquad \begin{bmatrix} 3 & x^2 \\ 2 & x^2 \end{bmatrix}$$

(A) 2 (B) ± 2 (C) (-2) (D) 4 (E) none of these [7] _____

8. Find the solution or solutions to the following system of linear equations.

$$\begin{aligned} 2x - 2y &= 1 \\ 2x + 4y &= 1 \\ 6x + 5y &= 3 \end{aligned}$$

(A) No solution (B) 2 distinct solutions (C) Exactly one solution (2,0)

(D) Exactly one solution $\left(\frac{1}{2}, 0\right)$ (E) There are infinitely many solutions

[8] _____

9. If one side of a square is doubled and the adjacent side is decreased by 2 feet, the area of the figure is increased by 96 square feet. Find the length of a side of the square.

(A) 24ft. (B) 12ft. (C) 8ft. (D) 10ft. (E) none of these [9] _____

10. Identify the point below which is <u>not</u> on the graph of $y = x^3 - 3x^2 + 2x - 1$

(A) $(-1, -7)$ (B) $(0, -1)$ (C) $(1, -1)$ (D) $(-1, 1)$ (E) none of these [10] _____

WCTM MATHEMATICS CONTEST, 1995

Test 5 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. Solve $(x-3)(x-4)=42$ and identify the true statement below.

(A) The product of the two solutions is 12.
(B) The product of the two solutions is -30.
(C) The sum of the two solutions is 91. [1] _____
(D) The sum of the two solutions is 0.
(E) none of the above.

2. Solve $x^4+36=13x^2$ and identify the true statement below:

(A) There are no positive solutions since all coefficients are positive.
(B) There are two solutions whose sum is 13.
(C) There are four solutions whose sum is -13. [2] _____
(D) There are four solutions whose sum is 0.
(E) none of the above.

3. Let $f(x)=x^2+3x-k$. If $f(k)=0$, identify the true statement below:

(A) k is necessarily zero.
(B) There are two possible values for k; their sum is 2.
(C) There are two possible values for k; their sum is -2. [3] _____
(D) k can be any non-zero number.
(E) none of the above

4. Let $f(x)=x^2-a^2$. Then $\dfrac{f(x+a)-f(x)}{x+a}$ simplifies to:

(A) $2+a$ (B) $2a$ (C) $x-a$ (D) $\dfrac{a(a+2x)}{a+x}$ (E) none of these [4] _____

5. $10^{-1}-5^{-2}$ is equal to:

(A) 0 (B) .06 (C) 5 (D) 15 (E) none of these [5] _____

6. $\dfrac{\sqrt{6}-3}{\sqrt{6}+9}$ is equal to:

(A) $-\dfrac{1}{3}$ (D) $\dfrac{4\sqrt{6}-11}{25}$

(B) $\dfrac{-33}{75}$ (E) none of the these [6] _____

(C) $3(2\sqrt{6}-7)$

7. For which value of k below will $x^2+kx+54$ and $x^2+kx-54$ both factor?

(A) 25 (B) 21 (C) 15 (D) 3 (E) none of these [7] _____

8. Express $\left(\dfrac{x}{3}+\dfrac{y}{5}\right)\left(\dfrac{y}{5}+\dfrac{x}{3}\right)$ as a single fraction in lowest terms. Which of the following does not appear in the final results?

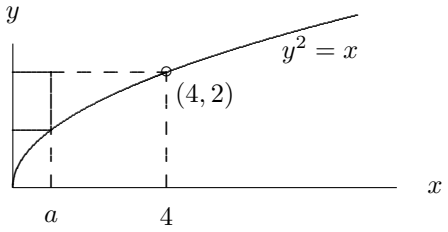
(A) $25x^2$ (B) 225 (C) $16xy$ (D) $9y^2$ (E) none of these [8] _____

9. Identify the true statement below.

(A) The number zero has no additive inverse.
(B) If $a<b$, then $\sqrt{(a-b)^2}=b-a$
(C) $(x+y)^n=x^n+y^n$ [9] _____
(D) If $a<0<b<c$, then $ab<ac$
(E) none of the preceding are true

10. In the diagram below, observe that for each number a such that $0<a<4$, a rectangle can be formed between the curve $x=y^2$ and $y=2$ as indicated. For a given number a, the area of the corresponding rectangle is expressed by:

(A) $2a-a\sqrt{a}$
(B) $2-a\sqrt{a}$
(C) $a\sqrt{a}$
(D) $(4-a)\sqrt{a}$
(E) none of the above



[10] _____

Grades HS
1995 Math Contest Exam

Exam	T1	T2	T3	T4	T5
P1	B	B	C	B	B
P2	D	E	B	D	D
P3	C	A	D	C	C
P4	C	C	C	A	D
P5	D	E	C	E	B
P6	A	C	C	D	D
P7	C	D	E	C	C
P8	B	E	D	D	C
P9	E	D	A	B	B
P10	E	C	D	D	A