WCTM MATHEMATICS CONTEST, 1993

Test 1 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. Succesive discounts of 10% and 20% are equivalent to a single discount of:

   (A) 30% (B) 15% (C) 72% (D) 28% (E) None of these [1] _____

2. The set of real solutions of $\frac{2}{3-x} = \frac{1}{3} - \frac{1}{x}$ is:

   (A) The empty set (B) $\{3\}$ (C) $\{-3\}$ (D) $\{3, -3\}$ (E) $\{0, 3\}$ [2] _____

3. A horizontal line is a common tangent to a circle whose center has coordinates (-5,-2) and to the parabola defined by $y = -2x^2 + 12x - 17$ at its vertex point. Find the radius of the circle.

   (A) 1 (B) 3 (C) 5 (D) 8 (E) Impossible to determine from given information [3] _____

4. In a group of sheep and blackbirds the number of heads is 20 less than one-half the number of legs. The number of sheep is:

   (A) 9 (B) 15 (C) 18 (D) 40 (E) None of these [4] _____

5. In a circle, the length of the radius is 10 inches and the length of a chord is 12 inches. At the intersections of the chord and the circle two lines are drawn tangent to the circle. Find the length of the line from one end of the chord to the intersection of the two tangent lines.

   (A) 7.5 inches (B) 8.5 inches (C) 8 inches
   (D) 7 inches (E) 6.5 inches
   [5] _____

6. Mr. Clark desires to lay a concrete slab over his driveway. Find the total cost, in dollars, for the concrete if the slab has dimensions 27 feet by 54 feet by 4 inches and the concrete costs $35 per cubic yard.

   (A) $720 (B) $630 (C) $480 (D) $560 (E) $575.25 [6] _____

7. A tin can with height = diameter is filled with a metal ball touching the sides and ends of the can, and the rest of the can is filled with water. What fraction of the volume of the can is taken up by the water?

   (A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}\pi$ [7] _____

8. Each student in a geometry class promised to contribute x cents towards a gift, costing \$15.00, for their teacher. However, at the end of the year, five students refused to pay their share, as they were upset about their grades, so the others had to pay 10 cents more than expected. The number of students in the class is a two digit number whose units digit is

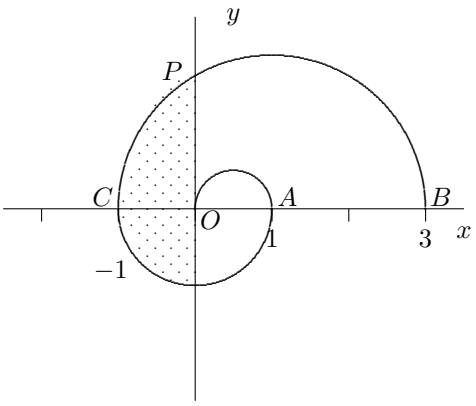
(A) 5 (B) 6 (C) 0 (D) 2 (E) 4 [8] _____

9. A spiral beginning at the origin is constructed by drawing semi-circles having diameters OA, CA, and CB as shown. The area of the shaded region, inside the spiral and left of the vertical axis, is:

(A) $\frac{2\pi}{3}$ (B) $\frac{5\pi - 3}{6}$ (C) $\frac{11\pi - 6\sqrt{3}}{12}$

(D) $\frac{4\pi - 2\sqrt{2} + 2}{3}$ (E) None of these

[9] _____



10. The expression $\frac{\sqrt{2}}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$ is equivalent to:

(A) $\frac{3 + \sqrt{6} + \sqrt{15}}{6}$ (B) $\frac{\sqrt{6} - 2 - \sqrt{10}}{6}$ (C) $\frac{2 + \sqrt{6} + \sqrt{10}}{10}$

(D) $\frac{2 + \sqrt{6} - \sqrt{10}}{6}$ (E) None of these

[10] _____

WCTM MATHEMATICS CONTEST, 1993

Test 2 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. Two chemicals are mixed with water to make 40 liters of spray:

Chemical	Amount Needed	Cost per Kilogram
A	3 Kilograms	\$1.50
B	2.5 Kilograms	\$1.00

Find the cost in dollars for buying chemicals used to spray 150 fruit trees, if it takes 2 liters of spray per tree.

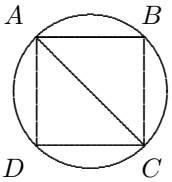
(A) \$52.50 (B) \$48.75 (C) \$51.75 (D) \$46.25 (E) None of these [1] _____

2. There are four pairs (x_i, y_i) $i = 1, 2, 3, 4$ which satisfy the system $x + y = \frac{9}{2}$ and $2x^2y^2 - 13xy + 18 = 0$. The sum $x_1 + x_2 + x_3 + x_4$ is equal to:

(A) -3 (B) 5 (C) 2 (D) 9 (E) 0 [2] _____

3. In the diagram, $ABCD$ is a square. If the length of AC is 3, find the area of the region lying outside the square but inside the circle.

(A) $\frac{9}{4}(\pi - 2)$ (B) $\frac{3}{2}(\pi - 1)$ (C) $\frac{9\pi - 3}{8}$ (D) $\frac{\sqrt{3}\pi}{2}$ (E) $\frac{3\pi\sqrt{2}}{5}$



[3] _____

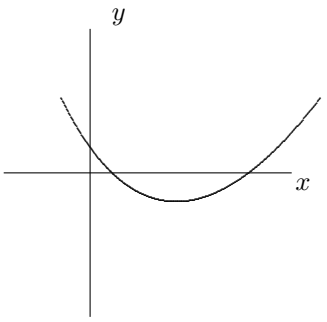
4. Find the number which exceeds its square by the maximum amount. The cube of this number minus the square of this number is:

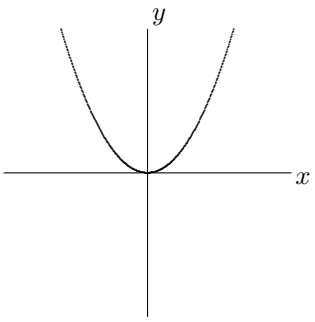
(A) $-\frac{1}{8}$ (B) - 2 (C) 4 (D) 2 (E) None of these [4] _____

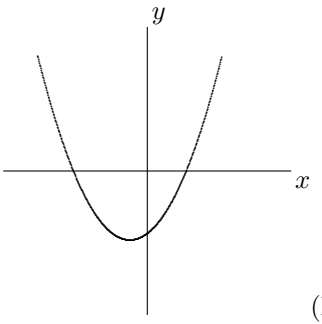
5. $\left(\frac{1}{2}\right)^4 + \left(\frac{1}{16}\right)^0 - (64)^{-1/2} - (-32)^{-4/5}$ equals:

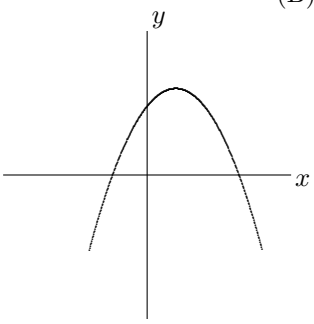
(A) $1\frac{13}{16}$ (B) 4 (C) 1 (D) $\frac{7}{8}$ (E) $\frac{1}{16}$ [5] _____

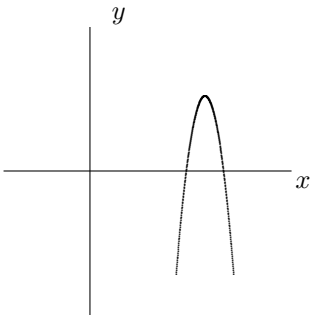
6. Which of the following could be the graph of $y = ax^2 + bx + c$ if a and b are positive integers and c is a negative integer.

(A) 

(B) 

(C) 

(D) 

(E) 

[6] _____

7. Let $|x|$ be x if x is not negative and $-x$ if x is negative. Then for the equation $|x|^2 + |x| - 6 = 0$,

(A) there is only one root (B) the sum of the roots is 1

(C) the sum of the roots is 0 (D) the product of the roots is 4 [7] _____

(E) the product of the roots is -6

8. A person from an audience selects any five letters from the alphabet. A mind reader then gets five tries to guess any one of these five letters. Assume selections by the person and the mind reader are random. What is the (approximate) probablity that the mind reader will not guess one of the five letters?

(A) 24% (B) 76% (C) 81% (D) 31% (E) 62% [8] _____

9. $n!$ means n factorial, and is defined to be n times $(n-1)$ times $(n-2)$ down to 3 times 2 times 1. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Find the number of zeros at the end of 10,000!, expressed in usual form.

(A) 2,499 (B) 2,000 (C) 2,400 (D) 2,399 (E) 3,299 [9] _____

10. A fish in a circular pond starts on an edge and swims north 60 feet to another point on the edge. He then swims due east for 80 feet to another point on the edge. What is the diameter of the pool?

(A) 80 feet (B) 92 feet (C) 100 feet (D) 1000 feet (E) 96 feet [10] _____

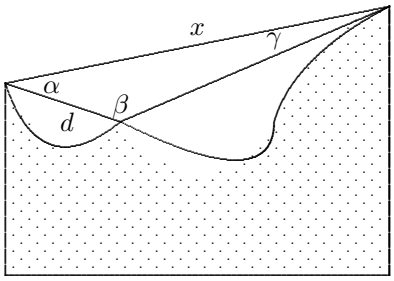
WCTM MATHEMATICS CONTEST, 1993

Test 3 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. The Corps of Engineers is planning to bridge a certain canyon. The distance "x" across the canyon must be determined. The surveyor is able to determine angles α and β and the distance "d" as illustrated in the figure. A formula for the distance "x" in terms of α, β and "d" is:



(A) $\dfrac{d\sin(\alpha+\beta)}{\sin\beta}$ (B) $\dfrac{d\sin\alpha}{\sin(\alpha+\beta)}$

(C) $\dfrac{d\sin\beta}{\sin(\alpha+\beta)}$ (D) $\dfrac{d}{\sin(\alpha+\beta)}$

(E) $\dfrac{d\sin\beta}{\sin\alpha+\sin\beta}$

[1] _____

2. Assume the three statements are true:
   I. All freshmen are human.
   II. All students are human.
   III. Some students think.

   Consider the following statements: (1) All freshmen are students. (2) Some humans think. (3) No freshmen think. (4) Some humans who think are not students. Which of these statements are logical consequences of I,II, and III?

(A) 2 (B) 4 (C) 2,3 (D) 2,4 (E) 1,2 [2] _____

3. Simplify the following using the properties of logarithms: $3\log_2 x - 2\log_2 m$

(A) $\log_2(x^3 - m^2)$ (B) $\log_2\left(\dfrac{x^3}{2m}\right)$ (C) $\log_2(x^3m^2)$

[3] _____

(D) $\log_2\left(\dfrac{x^3}{m^2}\right)$ (E) none of these

4. If $\log_2 x = 3.1, \log_2 y = 1.6$, and $\log_2 z = -2.7$ then $\log_2\left(\dfrac{xy}{2z^2}\right)$ is:

(A) $-.4$ (B) .7 (C) -6.1 (D) -1.7 (E) None of these [4] _____

5. Consider the following two relations: I. $y = x^2$ II. $x = y^2$

Find the true statement in the following:

(A) Both relations are functions of x (B) Neither relation is a function of x

(C) Only relation I is a function of x (D) Only relation II is a function of x

(E) None of these statements are true

[5] _____

6. A line has a slope of $-\frac{1}{2}$ and passes through (2,4). What is the slope-intercept form of the equation of the line?

(A) $y = -\frac{1}{2}x - 5$ (B) $y = -\frac{1}{2}x + 5$ (C) $y = \frac{1}{2}x - 3$

(D) $y = -\frac{1}{2}x - 3$ (E) $y = \frac{1}{2}x + 6$

[6] _____

7. If the perimeter of a rectangle is P and the length of a side is L, the area is:

(A) $\frac{P(P-2L)}{2}$ (B) $\frac{LP}{2}$ (C) $\frac{P(P-L)}{2}$ (D) $\frac{PL}{2} - L^2$ (E) $(P-2L)L$ [7] _____

8. Find the remainder when 3^{100} is divided by 7.

(A) 1 (B) 3 (C) 4 (D) 2 (E) 5 [8] _____

9. Suppose each of the three roots of $3x^3 - 24x^2 + 5x - 6 = 0$ is dimininshed by 2. The transformed equation becomes $3x^3 + ax^2 - 55x - 68 = 0$. Then a equals:

(A) 6 (B) -6 (C) 16 (D) -16 (E) 8 [9] _____

10. The formula for the velocity, v, of a liquid flowing through an opening is $v = \sqrt{2gh}$, where "g" is the acceleration due to gravity and "h" is the height of the liquid above the opening. If "h" is quadrupled, how is "v" changed?

(A) v is squared (B) v is quadrupled (C) v is doubled

(D) v is halved (E) v is increased by a factor of $\sqrt{2}$

[10] _____

WCTM MATHEMATICS CONTEST, 1993

Test 4 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. The endpoints of a diameter of a circle are $A(-5,-4)$ and $B(7,9)$. What is the equation of the circle?

   (A) $(x+1)^2+\left(y+\frac{5}{2}\right)^2=\frac{313}{4}$ (B) $(x-1)^2+\left(y-\frac{5}{2}\right)^2=\frac{313}{4}$

   (C) $(x-1)^2+\left(y-\frac{5}{2}\right)^2=\frac{29}{4}$ (D) $(x+1)^2+\left(y+\frac{5}{2}\right)^2=\frac{29}{4}$

   (E) None of these

   [1] _____

2. An automobile went up a hill at 10 m.p.h. and down the same distance at 20 m.p.h. The average speed for the round trip was:

   (A) $12\frac{1}{2}$ m.p.h. (B) $13\frac{1}{3}$ m.p.h. (C) $14\frac{1}{4}$ m.p.h. (D) 15 m.p.h. (E) None of these [2] _____

3. The area of a square inscribed in a semi-circle is to the area of a square inscribed in the entire circle as:

   (A) 1:2 (B) 2:3 (C) 2:5 (D) 3:4 (E) 3:5 [3] _____

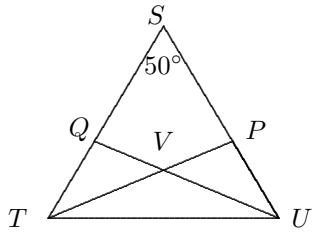
4. If the relationship between temperature in Celsius (C) and Farhenheit (F) is given by:

   $$C=\frac{5}{9}(F-32)$$

   find the temperature, in degrees Fahrenheit, which corresponds to 25 degrees Celsius.

   (A) 82 (B) 86 (C) 76.5 (D) 79 (E) None of these [4] _____

5. In the figure below, the length of TS and SU are equal and angle $S=50°$. If segments TP and UQ are angle-bisectors meeting at V, how many degrees are in angle TVU?

   (A) 132 (B) 115 (C) 125

   (D) 122.5 (E) 105

   [5] _____

6. Two equal parallel chords are drawn 8 inches apart in a circle of radius 8 inches. The area of the part of the circle between the chords equals:

(A) $21\frac{1}{3}\pi - 32\sqrt{3}$ (B) $32\sqrt{3} + 21\frac{1}{3}\pi$ (C) $32\sqrt{3} + 42\frac{2}{3}\pi$

(D) $16\sqrt{3} + 21\frac{1}{3}\pi$ (E) $42\frac{2}{3}\pi$

[6] _____

7. The sides of a triangle are of length a, b, and $c = \sqrt{a^2 + ab + b^2}$. Find its greatest angle, in degrees.

(A) 150 (B) 135 (C) 75 (D) 105 (E) 120 [7] _____

8. The rails on a railroad are 30 feet long. When the train passes over the point where the rails are joined, there is an audible click. The speed of the train in miles per hour is approximately the number of clicks heard in:

(A) 20 seconds (B) 2 minutes (C) $1\frac{1}{2}$ minutes

(D) 30 seconds (E) None of these

[8] _____

9. If $f(x) = \dfrac{x(x-1)}{2}$,then $f(x+2)$ equals:

(A) $f(x) + f(2)$ (B) $f(x)(x+2)$ (C) $x(x+2)f(x)$

(D) $\dfrac{xf(x)}{x+2}$ (E) $\dfrac{(x+2)f(x+1)}{x}$

[9] _____

10. The first three terms of the binomial expression $(a+bx)^n$ are $1 - \dfrac{2x}{9} + \dfrac{2}{27}x^2$; the next term is:

(A) $\dfrac{-2x^3}{81}$ (B) $\dfrac{-2x^3}{343}$

(C) $\dfrac{-20x^3}{729}$ (D) $\dfrac{-14x^3}{2187}$ (E) Not enough information is given

[10] _____

WCTM MATHEMATICS CONTEST, 1993

Test 5 **NAME**: _____

CLASS AB **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

1. A car is driven 20 m.p.h. up a steep slope one mile in length, and then continues down the other side, which is also one mile in length. What constant speed must the car be driven on the down slope, in order to average 40 m.p.h. for the entire two-mile stretch?

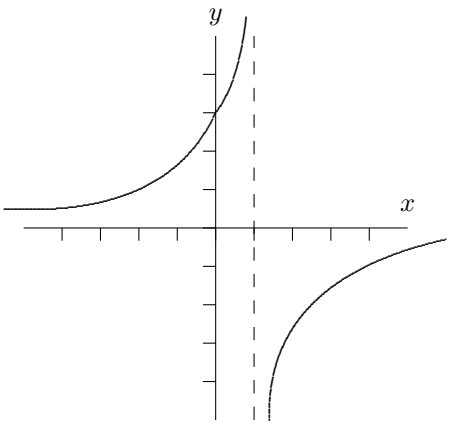
   (A) 60 m.p.h. (B) 80 m.p.h. (C) 64 m.p.h. (D) 120 m.p.h. (E) None of these [1] _____

2. The solution of the matrix equation $AX = B$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 12 \\ -13 \end{bmatrix}$ is:

   (A) $X = \begin{bmatrix} -14 \\ 49 \end{bmatrix}$ (B) $X = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$ (C) $X = \begin{bmatrix} -\frac{1}{6} & -\frac{1}{3} \\ -\frac{1}{2} & -\frac{1}{6} \end{bmatrix}$

   (D) $X = [12 \quad -26]$ (E) None of these

   [2] _____

3. 

   This is the graph of $y =$

   (A) $\dfrac{6-2x}{x-1}$ (B) $\dfrac{3-x}{(x-1)^2}$

   (C) $\dfrac{x-3}{\frac{1}{2}(x-1)}$ (D) $\dfrac{x-3}{x-1}$

   (E) $\dfrac{3}{1-x}$

   [3] _____

44. If $x < 0$, then $\left|x - \sqrt{(x-1)^2}\right|$ equals:

   (A) 1 (B) $1-2x$ (C) $-2x-1$ (D) $1+2x$ (E) $2x-1$ [4] _____

5. Each score for an exam will be computed by the formula $4R - W + 50$, where R denotes the number of right answers and W the number of wrong answers. Unanswered questions are not counted as being wrong. The maximum possible score is 250 and the minimum is 0, as there are 50 questions. The total number of different ways of receiving a score of 120 is:

(A) 4 (B) 3 (C) 7 (D) 5 (E) 6 [5] _____

6. Suppose the cost of producing clock radios can be approximated by

$$C(x) = 12x + 100$$

where $C(x)$ is the cost in dollars to produce x radios. What is the fixed cost in this formula?

(A) 12 (B) 100 (C) Not enough information to determine

(D) $\frac{100}{12}$ (E) $\frac{12}{100}$ [6] _____

7. If $y < 0$, then $xy + y > 0$ for all values of x such that:

(A) $x > 1$ (B) $x < -1$ (C) $-1 < x < 1$ (D) $x < 0$ (E) $x > -1$ [7] _____

8. A positive number n is tripled and increased by 7. The result is squared producing 1600. Find the units' digit of n in its base ten representation.

(A) 1 (B) 5 (C) 8 (D) 7 (E) 4 [8] _____

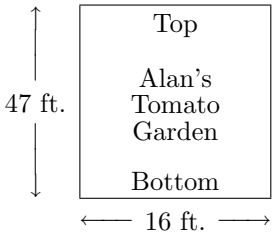
9. Which of the following is false as x increases?

(A) $f(x) = 2^x$ is increasing (B) $f(x) = 2^{-x}$ is decreasing

(C) $f(x) = \left(\frac{1}{10}\right)^x$ is decreasing (D) $f(x) = (\log_{10} 9)^x$ is increasing [9] _____

(E) $f(x) = \log_{10}(2^x)$ is increasing

10. Farmer Brown's son, Alan, desires to plant tomatoes in his rectangular garden, with parallel rows running from top to bottom. The row centers need to be 18 inches apart. The first plant in each row is to be 18 inches from the top edge of the plot, while the last plant need only be 6 inches from the bottom edge. The left row center and the right row center each need to be 6 inches from the side borders of the plot. How many tomato plants will be required if the plants are spaced 18 inches apart down each row?



(A) 450 (B) 341 (C) 300 (D) 452 (E) 332 [10] _____

Grades HS
1993 Math Contest Exam

Exam	T1	T2	T3	T4	T5
P1	D	A	C	B	E
P2	A	D	A	B	B
P3	B	A	D	C	E
P4	E	A	D	E	B
P5	A	D	C	B	C
P6	B	C	B	B	B
P7	B	C	D	E	B
P8	C	D	C	A	A
P9	C	A	B	E	D
P10	A	C	C	C	B