Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007

Test 1 **NAME**: _____

CLASS 9 & 10 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. Joan has the same number of sisters as she has brothers, but her brother Matt has twice as many sisters as he has brothers. If their family has fewer than 10 children, how many children are there?

   (A) 5 (B) 6 (C) 7 (D) 8 (E) 9 [1] _____

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2. On a line:

   - Point B is 1 unit to the right of point A.
   - Point C is 2 units to the left of point A.
   - Point D is 3 units to the right of point B.
   - Point E is 12 units to the left of point D.

   How many units is the distance from A to E?

   (A) 2 (B) 6 (C) 8 (D) 12 (E) 18 [2] _____

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3. Five paper bags contain a total of thirty apples.

   - The first and second bags contain a total of fourteen apples.
   - The second and third contain a total of ten apples.
   - The third and fourth contain a total of nine apples.
   - The fourth and the fifth contain a total of twelve apples.

   How many apples are in the fifth bag?

   (A) 4 (B) 5 (C) 6 (D) 7 (E) 8 [3] _____

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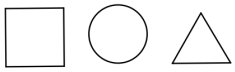
4. Two traffic lights turn red together at exactly 5:00 PM. One light is on a 36-second cycle from red back to red, and the other is on a 48-second cycle. At what time will they again turn red together? Give the answer exact to seconds.

   (A) 5:01:12 (B) 5:01:24 (C) 5:02:08 (D) 5:02:12 (E) 5:02:24 [4] _____

Go to back⟹

5. In a kindergarten class, each child is given a red crayon, a green crayon, a blue crayon, and the following picture:



Each child is then asked to color each of the shapes in the picture. They are told to color each figure with only one color, and no two figures have the same color. In how many different ways can the children color the picture?

(A) 3 (B) 4 (C) 6 (D) 9 (E) 12 [5] _____

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6. The following numbers are the first five terms in a pattern: 2, 6, 12, 20, 30. What is the tenth term?

(A) 34 (B) 38 (C) 42 (D) 90 (E) 110 [6] _____

---

7. If Jane has only dimes and nickels, in how many different ways can she make change for a dollar? Assume she always has enough coins to make the change.

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12 [7] _____

---

8. There is a least possible natural number value for k for which $k = \frac{n}{3} + \frac{n}{4}$, where n is a whole number. Multiply k by seven, and then sum the digits of the result.

(A) 3 (B) 10 (C) 12 (D) 13 (E) 17 [8] _____

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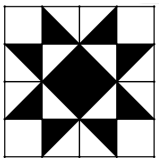
9. Consider the eight-point-star quilt block shown below. How many lines of symmetry does the block have?



(A) 0 (B) 1 (C) 2 (D) 4 (E) 8 [9] _____

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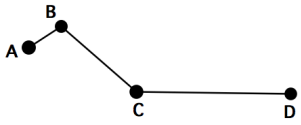
10. The distance from C to D is 24 miles. The distance from B to C is $\frac{2}{3}$ of the distance from C to D. The distance from A to B is $\frac{3}{8}$ of the distance from B to C. What is the distance from A to B?



(A) 4 (B) 6 (C) 9 (D) 16 (E) 46 [10] _____

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007

Test 2 **NAME**: _____

CLASS 9 & 10 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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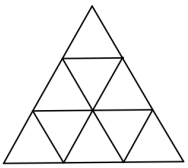
1. We will define an isosceles trapezoid as a quadrilateral, or four-sided polygon, with exactly one set of parallel-sides and a pair of congruent base angles. How many different isosceles trapezoids can you count in the figure below?



(A) 3 (B) 12 (C) 15 (D) 18 (E) 21 [1] _____

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2. Bekah has three brass house number digits: 2, 3, and 5. How many distinct numbers can she form using one or more of the digits?

(A) 6 (B) 7 (C) 9 (D) 12 (E) 15 [2] _____

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3. Find the value of the expression below.

$$\frac{1}{4+\frac{1}{4+\frac{1}{4\frac{1}{4}}}}$$

(A) $\frac{1}{4}$ (B) $\frac{4}{17}$ (C) $\frac{17}{72}$ (D) $\frac{72}{288}$ (E) $\frac{72}{305}$ [3] _____

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4. A player's season-home-run total is assumed to be proportional to any mid-season total. After 108 games in 1999, Mark McGwire had accumulated 42 home runs. At that point, what was his projected season-home-run total? There were 162 games are in the 1999 season.

(A) 23 (B) 63 (C) 75 (D) 87 (E) 91 [4] _____

---

5. Determine a value of k so that $2x+1$ is a factor of $2x^2+7x+k$.

(A) -4 (B) -3 (C) 1 (D) 2 (E) 3 [5] _____

Go to back⟹

6. A speedboat costs $35,000. The salesman says that the boat will *depreciate* in value by 10% each year. In other words, each year the boat will be worth 10% less than the year before. How many years must pass before the boat is worth only half of the original price?

(A) 4.6 (B) 5 (C) 6.6 (D) 7.2 (E) 8.3 [6] _____

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7. The pups dug into dinner. Polaris ate more than Sirius. Castor ate less than Pollux. Pollux ate less than Sirius, but more than Rigel. Which pup ate the second most?

(A) Castor (B) Polaris (C) Pollux (D) Rigel (E) Sirius [7] _____

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8. The Ice Cream Shop offers five different flavors daily. Today's flavors include vanilla, chocolate, strawberry, cookies and cream, and raspberry. Homemade waffle, pretzel, and sugar cones are also available. If Jane orders a double-dip cone, from how many different two flavor ice cream cones can she choose? Make your count assuming that the order of the dips doesn't matter to Jane. She would consider a raspberry on the bottom with vanilla on top the same as having vanilla under a raspberry dip.

(A) 15 (B) 30 (C) 45 (D) 60 (E) 75 [8] _____

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9. Jessie's math teacher assigned one homework problem Monday night. On each of the following days for the rest of the week, she assigned three times as many problems as on the previous night. At the end of the week, all the problems from the chapter had been assigned. How many problems did the chapter have?

(A) 13 (B) 15 (C) 63 (D) 81 (E) 121 [9] _____

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10. Which of the following statements is true for the data set below?

$$12, 5, 15, 13, 6, 9, 3, 5, 7, 5$$

(A) mode $<$ median $<$ mean
(B) mode $>$ median $>$ mean
(C) mode $=$ median $=$ mean
(D) mode $<$ mean $<$ median
(E) mode $>$ mean $>$ median [10] _____

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007

Test 3 **NAME**: _____

CLASS 9 & 10 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. If P represents the product of all prime numbers less than 1000, what is the value of the units digit of P ?

   (A) 0 (B) 1 (C) 2 (D) 6 (E) 8 [1] _____

---

2. If it is 11 A.M. right now, what time will it be 2007 hours from now?

   (A) 12:00 AM (B) 12:00 PM (C) 1:00 AM (D) 2:00 AM (E) 3:00 AM [2] _____

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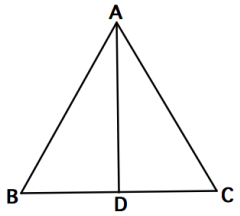
3. Triangle ABC is equilateral, and $\overline{AD}$ is an altitude. If the length of $\overline{AB}$ is 12, find the distance from B to the midpoint of $\overline{AD}$.



   (A) 3 (B) 6 (C) $3\sqrt{3}$ (D) $3\sqrt{5}$ (E) $3\sqrt{7}$ [3] _____

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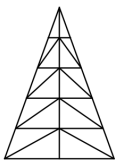
4. How many triangles are in the figure below?



   (A) 7 (B) 18 (C) 23 (D) 35 (E) 43 [4] _____

---

5. The mean of a set of numbers is 120. If one number in the set is increased by 300, the mean increases to 135. How many numbers are in the set?

   (A) 12 (B) 15 (C) 18 (D) 20 (E) 24 [5] _____

Go to back⟹

6. How many solutions does the equation $x = 4\cos(x)$ have if $-5 \le x \le 5$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 [6] _____

7. What is the units digit in the decimal representation of 7^{2007} ?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9 [7] _____

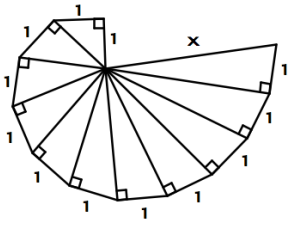
8. George had a number of jelly beans and ate one-third of them. Martha found the remaining jelly beans and ate some until only one-fifth of the original number of jelly beans remained. What fraction of the jelly beans that Martha found did she eat?

(A) $\frac{2}{3}$ (B) $\frac{7}{15}$ (C) $\frac{3}{5}$ (D) $\frac{7}{10}$ (E) $\frac{3}{10}$ [8] _____

9. Suppose that $4x - 9 \le f(x) \le x^2 - 4x + 7$ for $x \ge 0$. Find $\lim_{x \to 4} f(x) =$

(A) -9 (B) 0 (C) 1 (D) 7 (E) doesn't exist [9] _____

10. Determine the length of the segment labeled x in the figure below.



(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{10}$ (D) $2\sqrt{2}$ (E) $3\sqrt{3}$ [10] _____

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007

Test 4 **NAME**: _____

CLASS 9 & 10 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. What number is doubled when two-thirds of it is added to 36?

   (A) 18 (B) 24 (C) 27 (D) 30 (E) 36 [1] _____

---

2. A boy is 160 centimeters tall. If he could walk around the earth along the equator, the top of his head would travel further than his feet. Estimate how much further (in meters) his head would travel. Treat the Earth as a sphere with radius 6378 kilometers.

   (A) 100 (B) 10 (C) 400 (D) 4000 (E) 40,000 [2] _____

---

3. The sum of five consecutive integers is 5^3 . Find the product of the smallest and largest of them.

   (A) 500 (B) 616 (C) 621 (D) 625 (E) 750 [3] _____

---

4. A bicycle wheel has a 12 inch radius. Approximately how many revolutions are required to travel fives miles on this bicycle?

   (A) 840 (B) 2100 (C) 3600 (D) 4200 (E) 8400 [4] _____

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5. A gambler has two coins in his pocket – one fair coin and one two-headed coin. He selects at random one of these coins, and then flips the coin twice. If he gets two heads, what is the probability that he selected the fair coin?

   (A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$ [5] _____

---

6. Find the sum of the coefficients in the expansion of $(a+b+c)^8$.

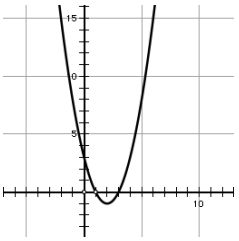
   (A) 3 (B) 24 (C) 27 (D) 256 (E) 6561 [6] _____

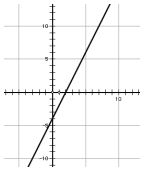
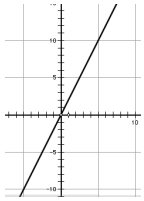
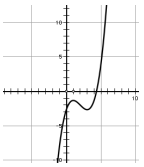
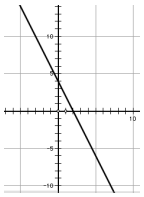
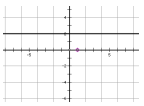
Go to back⟹

7. Of ten boxes, five contain pencils, four contain pens, and two contain both pens and pencils. How many boxes contain neither pens nor pencils?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 [7] _____

8. The function $f(x)$ is given in the sketch below. Select the sketch of $f'(x)$.

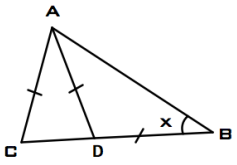


(A)  (B)  (C)  (D)  (E)  [8] _____

9. Consider a circle with a chord 5 units from the center. If the length of that chord is 10 units, then how long is the diameter?

(A) 10 (B) $10\sqrt{2}$ (C) 15 (D) 5 (E) $5\sqrt{5}$ [9] _____

10. In the triangle below, $\overline{AB} = \overline{CB}$ and $\overline{AC} = \overline{AD} = \overline{DB}$. Determine the measure of the angle x.



(A) 18° (B) 30° (C) 36° (D) 64° (E) 72° [10] _____

Wolsborn-Drazovich STATE MATHEMATICS 51st CONTEST, 2007

Test 5 **NAME**: _____

CLASS 9 & 10 Grade **SCHOOL**: _____

SCORING: *20 points for each correct answer, -5 for each wrong answer.*

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1. The sum of the second and the fifth terms of an arithmetic sequence is equal to the sixth term of the sequence. If the sixth term is 25, find the tenth term of the sequence.

   (A) 30 (B) 40 (C) 45 (D) 50 (E) 55 [1] _____

---

2. Let $f(x) = \sin(x)$ and let h be the smallest positive real number so that the graph of $y = f(x-h)$ will be symmetric about the y-axis. Find $\sin(h)$.

   (A) 1 (B) -1 (C) 0 (D) $-\frac{2}{\sqrt{2}}$ (E) $\frac{2}{\sqrt{2}}$ [2] _____

---

3. Anna says that a fifty-five percent chance exists that she will go to the movie tomorrow if it is raining at noon, and a thirty percent chance she'll go if it is not raining. Willard forecasts a forty percent chance of rain at noon. On the basis of these numbers, what is the probability that Anna will go to a movie?

   (A) 12% (B) 18% (C) 22% (D) 40% (E) 70% [3] _____

---

4. What is the maximum number of intersection points when two circles and three straight lines intersect each other? Assume that the figure is drawn in a plane and that no figure coincides with another.

   (A) 12 (B) 15 (C) 16 (D) 17 (E) 18 [4] _____

---

5. If $\log_b(xy) = 11$ and $\log_b\left(\frac{x}{y}\right) = 5$, what is $\log_b x$?

   (A) 6 (B) 8 (C) 16 (D) 32 (E) $\frac{11}{5}$ [5] _____

---

6. Consider the function $f(x)$ defined on the interval $[0,1]$ with all of the following properties:

   (a) $f(x)$ is continuous on $[0,1]$.

   (b) $f(x)$ is decreasing on the entire interval $[0,1]$.

   (c) $f(0.1) = 1, f(0.3) = 0.5, f(0.7) = -0.1$ and $f(0.9) = -0.5$.

   According to the Intermediate Value Theorem, which one value might solve $f(x) = 0$?

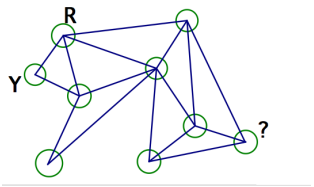
   (A) $x = 0.2$ (B) $x = 0.3$ (C) $x = 0.5$ (D) $x = 0.8$ (E) none of these [6] _____

Go to back⟹

7. Daddy Warbucks was a cartoon character started in 1924. He was called a tycoon, and has come to symbolize people with lots of money. Suppose that Daddy Warbucks is very methodical, and knows just how many \$100, \$500 and \$1 bills he's carrying. The bigger denominations are for his impulse buys. He carries a specific number of \$1 for tips. If he has 500 bills total in his briefcase, and their total value is \$50,000, how many \$1 bills must he be carrying?

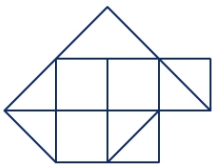
(A) 100 (B) 200 (C) 300 (D) 400 (E) 500 [7] _____

8. Consider the graph below. Each circle at a vertex is colored red, yellow or blue. Once colored, no two vertices of a triangle are colored the same color. What is the color of the vertex marked with the question mark?



(A) red (B) yellow (C) blue (D) red or yellow (E) blue or yellow [8] _____

9. It is impossible to draw the following design without lifting your pencil from the page, traversing each and every edge precisely once. However, if you are permitted to trace over a small number of edges twice in order to trace over the entire figure, what is the least number of reused edges needed?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 [9] _____

10. On a certain test, the average score for the women in the class is 83, whereas the average score for the men is 71. If the average score of all students is 80, what percent of the students are women?

(A) 25% (B) $33\frac{1}{3}\%$ (C) 50% (D) $66\frac{2}{3}\%$ (E) 75% [10] _____

Grades 9-10
2007 Math Contest Exam Answer Key

Exam	T1	T2	T3	T4	T5
P1	c	c	a	c	c
P2	c	e	d	b	a
P3	d	e	e	c	d
P4	e	b	e	d	d
P5	c	e	d	c	b
P6	e	c	d	e	c
P7	d	e	b	c	d
P8	d	b	d	a	b
P9	d	e	d	b	b
P10	b	a	a	c	e