

Wolsborn-Drazovich STATE MATHEMATICS CONTEST, 2006
50th Anniversary



Test 1

NAME: _____

CLASS 9 & 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Ralph has a jar that contains seven pennies, two nickels, three dimes, and five quarters. If he randomly removes coins from the jar, how many must he take out to guarantee that he will have enough pennies and nickels to make nine cents in change?

(a) 5 (b) 6 (c) 8 (d) 14 (e) 16 [1] _____

2. A unit fraction is a proper fraction with a numerator of 1. There are two ways to express $\frac{4}{5}$ as a sum of three different unit fractions. Which of the following could be used together with $\frac{1}{20}$?

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$ (e) $\frac{1}{8}$ [2] _____

3. Study the following table for a pattern between the input values x and output values y . Applying the same rule, find the output value be for an input of $x = 18$.

x	36	53	43	23	942	18
y	8	7	6	4	14	

(a) 2 (b) 3 (c) 5 (d) 8 (e) 9 [3] _____

4. Seventy-five students have just returned from a class trip to Washington D.C. The students visited the following sites. How many students visited none of the three sites?

- Twenty students visited the Washington Monument.
- Thirty-five students visited the White House.
- Fifteen students visited the Kennedy Center.
- Ten students visited the Kennedy Center and the White House.
- Eight students visited the White House and the Washington Monument.
- Six students visited the Washington Monument and the Kennedy Center.
- Four students visited all three sites.

(a) 15 (b) 10 (c) 25 (d) 29 (e) 50 [4] _____

Go to back⇒

5. Three solid balls, each of radius 3.25 cm, are stored in a cylindrical can with the smallest possible radius and volume. What fraction of the can's volume is air?

- (a) $\frac{2}{3}$ (b) $\frac{2}{9}$ (c) $\frac{7}{9}$ (d) $\frac{1}{2}$ (e) $\frac{1}{3}$ [5] _____
-

6. My son is five times as old as my daughter, and my wife is five times as old as my son. I am twice as old as my wife, and my grandmother, who is as old as all of us put together, is 81 years old today. What is the sum of the ages of my son and wife?

- (a) 25 (b) 26 (c) 28 (d) 30 (e) 35 [6] _____
-

7. A student's score on the first four Inter-school Math Contests were 5, 4, 3, and 3. What score does she need on the fifth and final contest to finish with a mean score of 4 on the five contests?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5 [7] _____
-

8. Suppose that $\triangle x$ means $x + 3$, that $\square x$ means x^2 , and that $\bigcirc x$ means $x + 1$. Find the value of the expression below.

$$\square \bigcirc 3 + \bigcirc \square 3 + \bigcirc \triangle 3$$

- (a) 35 (b) 33 (c) 31 (d) 16 (e) 9 [8] _____
-

9. Emma, Louise, Will, and Carlos each have two favorite hobbies that include collections. The collections are seashells, stamps, baseball cards, coins, comic books, dolls, bugs, and rocks. No two students collect the same things. From the clues below, decide which person collects stamps.

- Emma always finds things for both of her collections outdoors.
- One of Carlos's friends enjoys collecting coins.
- The person who collects comic books does not collect baseball cards.
- One of Carlos's hobbies involves a lot of reading.
- One girl collects dolls.
- Louise's family has a beach house, the perfect place for one of her collections.

- (a) Carlos (b) Emma (c) Louise (d) Will (e) insufficient information [9] _____
-

10. A trapezoid is inscribed in a circle of diameter 6. One base is the diameter of the circle. The other base has length 3. What is the perimeter of the trapezoid?

- (a) 12 (b) 13 (c) 15 (d) $9 + 2\sqrt{2}$ (e) $9 + 2\sqrt{3}$ [10] _____
-

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Test 2

NAME: _____

CLASS 9 & 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The average age of the 40 members of a computer science camp is 17 years. Of those attending, 20 are girls, 15 are boys, and 5 are adults. If the average age of the girls is 15, and the average age of the boys is 16, what is the average age of the adults?

(a) 30 (b) 28 (c) 27 (d) 24 (e) 21 [1] _____

2. What is the approximate speed in miles per hour of a race car that traverses a football field 100 yards long in one second?

(a) 360 mph (b) 36 mph (c) 34 mph (d) 205 mph (e) 60 mph [2] _____

3. Let A, B and C represent different positive digits. Consider the expression below, noting that ABC signifies a 3-digit number where A is the hundreds digit, B is the tens digit, and C is the ones digit.

$$\frac{ABC}{A + B + C}$$

Which of the following statements best describes its minimum value m ?

(a) $m < 3$ (b) $3 \leq m < 6$ (c) $6 \leq m < 9$ (d) $9 \leq m < 12$ (e) $m \geq 12$ [3] _____

4. Four dimes and four pennies are randomly placed in a row. What is the probability that the first and last coins are both pennies?

(a) $\frac{1}{8!}$ (b) $\frac{6!}{8!}$ (c) $\frac{1}{12}$ (d) $\frac{1}{70}$ (e) $\frac{3}{14}$ [4] _____

5. The point (a, b) is reflected over the y -axis to the point (c, d) , which is then reflected over the x -axis to the point (e, f) . Compute the value of $ab - ef$.

(a) $2ab$ (b) $ab + b^2$ (c) -1 (d) 0 (e) 1 [5] _____

Go to back \Rightarrow

6. Hiking boots that cost \$150 a pair are reduced by 40% for a special weekend sale. By what percent must the special sales price be increased to bring the price back to \$150?

- (a) 40% (b) 60% (c) $66\frac{2}{3}\%$ (d) 100% (e) $166\frac{2}{3}\%$ [6] _____
-

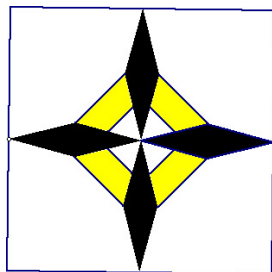
7. Consider the four positive integers 1, 3, 120 and n . If you multiply any two of them the product is one less than a perfect square. Select the best statement about the smallest possible value of n .

- (a) $n \leq 3$ (b) $3 < n \leq 8$ (c) $n = 15$ (d) $n = 21$ (e) $n = 33$ [7] _____
-

8. The faces of a cube are marked with the numbers 1 through 6. Each corner of the cube is assigned a *vertex number* that is equal to the sum of the numbers on the faces that meet at that vertex. What is the sum of all the vertex numbers?

- (a) 21 (b) 36 (c) 42 (d) 84 (e) 168 [8] _____
-

9. The figure below represents one quilt block of a design called “The Beautiful Star”. How many different **types** of convex polygons can be found in the design? Don’t forget to count the outer block shape.



- (a) 9 (b) 6 (c) 5 (d) 4 (e) 3 [9] _____
-

10. The following frequency table contains the test scores for Math 100. What is the **median** exam score?

Score	1	6	7	8	9	10	11	12	13	14	15	16	24
Frequency	1	1	2	6	10	16	13	9	8	5	2	1	1

- (a) 13 (b) 12 (c) 11 (d) 10 (e) 9 [10] _____
-

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Test 3

NAME: _____

CLASS 9 & 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Convert the base 8 number 146_8 to base 3.

- (a) 10210_3 (b) 1210_3 (c) 121_3 (d) 102_3 (e) 12_3 [1] _____
-

2. If $f'(a) = 0$ at a certain value a , which one statement concerning $f(a)$ is correct?

- (a) $(a, f(a))$ is a point at which the tangent line is parallel to the y -axis.
(b) $(a, f(a))$ is a point at which the tangent line is parallel to the x -axis.
(c) $(a, f(a))$ is a local maximum.
(d) $(a, f(a))$ is a local minimum.
(e) $(a, f(a))$ is a point of inflection. [2] _____
-

3. For what value of k is the following statement true?

$$\sqrt{\frac{a}{b} \sqrt{\frac{b}{a} \sqrt{\frac{a}{b}}}} = \left(\frac{a}{b}\right)^k$$

- (a) $-\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) $\frac{3}{8}$ (e) $-\frac{1}{2}$ [3] _____
-

4. Three identical balls are marked 1, 2, and 3 and placed in an urn. One ball is drawn, its number is recorded, and then the ball is returned to the urn. After repeating the process twice more, the sum of the numbers is recorded as 6. What is the probability that the ball numbered 2 was drawn in all three tries?

- (a) $\frac{1}{10}$ (b) $\frac{1}{9}$ (c) $\frac{1}{7}$ (d) $\frac{7}{9}$ (e) $\frac{7}{27}$ [4] _____
-

5. Describe the real numbers that comprise the domain of the real-valued function $f(x) = \sqrt{5 - \sqrt{x}}$.

- (a) $x \geq 0$ (b) $x \geq 25$ (c) $x \leq 25$ (d) $0 \leq x \leq \sqrt{5}$ (e) $0 \leq x \leq 25$ [5] _____
-

6. Which one of the following properties is true about the solutions of $2x^2 + 18 = 0$?

- (a) The product of the solutions is positive.
(b) The product of the solutions is zero.
(c) The product of the solutions is negative.
(d) The product of the solutions is an imaginary number.
(e) None of the above. [6] _____
-

Go to back \Rightarrow

7. A square has lines drawn in it that subdivide its interior. First, the diagonals are drawn. Lines are also drawn so that midpoints of the adjacent sides are connected. Determine the number of triangles contained in the figure.

- (a) 10 (b) 12 (c) 14 (d) 16h(e) 20 [7] _____

8. A set of cards is labeled with integers 1 to 200 and put into a bag. What is the probability that a card labeled with a number divisible by either 2 or 5 could be drawn?

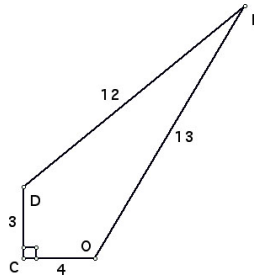
- (a) $\frac{1}{2}$ (b) $\frac{140}{200}$ (c) $\frac{120}{200}$ (d) $\frac{40}{200}$ (e) $\frac{20}{200}$ [8] _____

9. Although the Pacific Ocean, the Baltic Sea, and the Arctic Ocean are the only three bodies of water, they add up to the seven Cs. Substitute a different digit for each letter in the following statement. Note that the number 1 is not used. Which of the following statements about the letters used is **true**?

$$\begin{array}{r}
 P A C I F I C \\
 B A L T I C \\
 + A R C T I C \\
 \hline
 C C C C C C C
 \end{array}$$

- (a) $C = 5$ (b) $C = 9$ (c) $I = 7$ (d) $F = 8$ (e) $A = 4$ [9] _____

10. Find the area of the quadrilateral COLD drawn below.



- (a) 4 (b) 6 (c) 9 (d) 24 (e) 36 [10] _____

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Test 4

NAME: _____

CLASS 9 & 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Simplify the following expression so that each variable appears only once and none has a negative exponent:

$$\frac{a^{-5}b^{-9}c^7d^4e^{-2}}{b^{-14}c^{-3}d^7e^4f^{-3}}$$

- (a) $\frac{-a^5c^4}{b^5d^3e^2f^3}$ (b) $\frac{a^5c^4}{b^5d^3e^2f^3}$ (c) $\frac{b^5c^{10}f^3}{a^5d^3e^6}$ (d) $\frac{b^5c^4f^3}{a^5d^3e^2}$ (e) $\frac{-a^5b^5c^4}{d^3e^2f^3}$ [1] _____
-

2. Three dart players threw simultaneously at a tic-tac-toe board, each hitting a different square. What is the probability that the three hits constituted a win at tic-tac-toe?

- (a) $\frac{1}{3}$ (b) $\frac{4}{25}$ (c) $\frac{8}{27}$ (d) $\frac{2}{19}$ (e) $\frac{2}{21}$ [2] _____
-

3. Find the area of the region enclosed by the graph of $|y| + |2x| = 6$.

- (a) 24 sq units (b) 36 sq units (c) 48 sq units (d) 72 sq units (e) 18 sq units [3] _____
-

4. The 120 permutations of WDSMC are arranged in dictionary order, as if each were an ordinary five-letter word. Find the last letter of the 86th word in the list.

- (a) W (b) D (c) S (d) M (e) C [4] _____
-

5. Find the slope of the tangent line to the curve $f(x) = x^2 - 3x - 1$ at the point $\left(\frac{1}{2}, -\frac{9}{4}\right)$.

- (a) -2 (b) -1 (c) 0 (d) 1 (e) 2 [5] _____
-

6. If $(4, 7)$ is a point on the graph of $y = f(x)$, which of the following points is the corresponding point on the graph of $g(x) = 3f(2(x + 3)) - 5$?

- (a) $(5, 16)$ (b) $(11, -3)$ (c) $(-1, -3)$ (d) $(-1, 16)$ (e) $(5, -3)$ [6] _____
-

Go to back \Rightarrow

7. Solve the inequality $mx + n < k$ for x . Assume that $m < 0$.

- (a) $x < \frac{k-n}{m}$ (b) $x > \frac{k-n}{m}$ (c) $x > \frac{n-k}{m}$ (d) $x > k - n - m$ (e) $x < k - n - m$ [7] _____
-

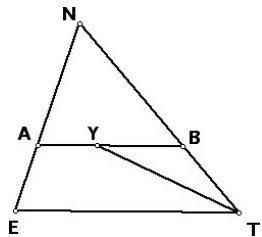
8. A culture of 100 bacteria doubles in number every two hours. How many hours will it take for the number of bacteria to reach 3200?

- (a) 32 (b) 16 (c) 12 (d) 10 (e) 8 [8] _____
-

9. Which one of the following functions **IS NOT** periodic with period 1 and amplitude 1?

- (a) $y = \cos(2\pi x - 1)$ (b) $y = \cos(2\pi x) + 3$ (c) $y = \sin(2\pi x - e) + 1$ [9] _____
(d) $y = \sin(2\pi x)$ (e) $y = \cos(2\pi x) + \sin(2\pi x)$
-

10. Given: $\overline{AB} \parallel \overline{ET}$; \overline{TY} bisects $\angle BTE$; $m\angle ANB = 60^\circ$; and $m\angle NAB = 70^\circ$. Find $m\angle BYT$.



- (a) 40° (b) 35° (c) 30° (d) 25° (e) 20° [10] _____
-

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Test 5

NAME: _____

CLASS 9 and 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. The two diagonals of a rhombus measure 10 and 24. What is its perimeter?

- (a) 26 (b) 44 (c) 52 (d) 68 (e) 120 [1] _____
-

2. The slope of the line $y = 2$ is

- (a) 0 (b) -2 (c) 2 (d) 1 (e) Undefined [2] _____
-

3. A drawer contains 100 red socks, 80 green socks, 60 blue socks, and 40 black socks. Bo selects socks one at a time from the drawer, but is unable to see the colors of the socks. What is the smallest number of socks that must be selected to **guarantee** that the selection contains at least 10 matching pairs?

- (a) 20 (b) 21 (c) 22 (d) 23 (e) 80 [3] _____
-

4. It's true that $\ln(x + 3) \geq 0$ if

- (a) $-3 < x < -2$ (b) $x \geq 1$ (c) $x \geq -2$ (d) $x > 0$ (e) $x \geq 4$ [4] _____
-

5. Given that $\cos x = \frac{1}{2}$ and that $-90^\circ < x < 90^\circ$, which one of the following has the largest value?

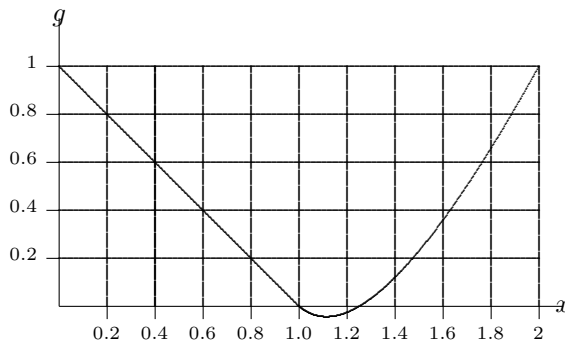
- (a) $\sin x$
(b) $\cos x$
(c) -2
(d) Answers (a), (b) & (c) all equal.
(e) Impossible to tell. [5] _____
-

6. If the amplitude of $y = \left(\frac{1}{k}\right) \cos(k^2\theta)$ is 2, then the period must be

- (a) π (b) 2π (c) 4π (d) 8π (e) 16π [6] _____
-

Go to back \Rightarrow

7. The graph below is that of $y = g(x)$. Compute $g(g(2))$.



- (a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (e) 0 [7] _____
-

8. Which one of the following statements is **true** about the solutions of $4x^2 + 6x + 9 = 9 - 2x$?

- (a) The sum of the solutions is less than their product.
(b) The sum of the solutions is equal to their product.
(c) The sum of the solutions is greater than their product.
(d) The product of the solutions is a complex number.
(e) None of these. [8] _____
-

9. Notice that the vowels in the word FACETIOUS are in alphabetic order. Consider all the possible new words made from rearrangements of the letters in this word. In how many of them are the vowels in alphabetic order?

Note: $C(n, r) = \frac{n!}{r!(n-r)!}$

- (a) $C(9, 5)$ (b) $C(9, 5) \cdot 4!$ (c) $C(9, 4)$ (d) $C(9, 4) \cdot 5!$ (e) $4! \cdot 5!$ [9] _____
-

10. The integral $\int_a^b x^2 e^{x^3} dx$ is equivalent to which one of the following integrals?

- (a) $\frac{1}{3} \int_{a^3}^{b^3} e^u du$ (b) $\int_{a^3}^{b^3} e^u du$ (c) $\frac{1}{3} \int_a^b e^u du$ [10] _____
(d) $-\frac{1}{3} \int_a^b e^u du$ (e) $3 \int_{a^3}^{b^3} e^u du$
-

Grades 9-10
2006 Math Contest Exam

Exam	T1	T2	T3	T4	T5
P1	e	b	a	c	c
P2	b	d	b	e	a
P3	d	d	d	b	d
P4	c	e	c	b	c
P5	e	d	e	a	e
P6	d	c	a	d	d
P7	e	b	e	b	e
P8	b	d	c	d	a
P9	a	b	a	e	b
P10	c	c	e	d	a