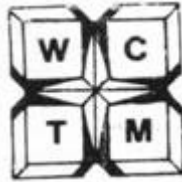


Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 1

NAME: _____

CLASS 9 & 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

1. Find the value of the expression $15 - 14 + 13 - 12 + 11 - 10 + 9 - 8 + 7 - 6 + 5 - 4 + 3 - 2 + 1$.

- (A) -1 (B) 8 (C) 10 (D) 12 (E) 16 [1] _____

2. Friends go to a party. At the first doorbell ring, one guest arrives. At the second ring, two more guests arrive than on the first ring. At the third ring, two more guests arrive than on the second ring, and so on. How many guests have arrived at the party after the eighth ring?

- (A) 15 (B) 36 (C) 49 (D) 64 (E) 81 [2] _____

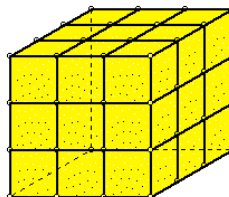
3. How many subsets of the set $\{a, b, c, d, e\}$ have fewer than two elements?

- (A) 1 (B) 5 (C) 6 (D) 10 (E) 26 [3] _____

4. A family has three daughters. The product of the daughters' ages is 200. The oldest daughter is twice the age of the middle daughter. What is the sum of their ages?

- (A) 18 (B) 20 (C) 22 (D) 27 (E) 31 [4] _____

5. You are given a cube, three inches on a side. This cube is painted, and then cut up into twenty-seven one-inch cubes. Some of the smaller cubes will have three painted faces; some two painted faces; some one and some none at all. Suppose you take these twenty-seven cubes and put them into a large paper bag, shake them up so that they are well-mixed, and then you reach inside the bag and pick one cube. What is the probability that this cube you picked will have paint on exactly two faces?



- (A) $\frac{0}{27}$ (B) $\frac{6}{27}$ (C) $\frac{8}{27}$ (D) $\frac{12}{27}$ (E) $\frac{18}{27}$ [5] _____

-
6. A rumor starts by someone telling the rumor to two people on May 1. Each of these people is responsible for telling the rumor to two others, which they do on May 2. So, on May 2, seven people know the rumor. On May 3, the four who heard it on May 2 each tells two people. The rumor process continues like this each day. On what day will 256 new people be told the rumor?



- (A) May 12 (B) May 11 (C) May 10 (D) May 9 (E) May 8 [6] _____
-

7. To the nearest whole number, how many miles an hour is a car traveling if the wheels have diameters of three feet and turn 400 times a minute?

- (A) 128 (B) 43 (C) 40 (D) 21 (E) 17 [7] _____
-

8. The units digit, or the ones digit, of the product of any six consecutive positive whole numbers always equals which number?

- (A) 8 (B) 6 (C) 4 (D) 2 (E) 0 [8] _____
-

9. The length of a rectangle is increased by 20 percent and its width is increased by fifty percent. By what percent does the area increase?

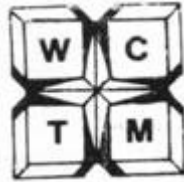
- (A) 70 (B) 80 (C) 100 (D) 170 (E) 180 [9] _____
-

10. Below is a table of grades a class scored on a given test. How many students scored 70 or above?

Scores	Frequency
90-99	3
80-89	5
70-79	12
60-69	5
under 60	2

- (A) 5 (B) 8 (C) 12 (D) 20 (E) 27 [10] _____
-

Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 2

NAME: _____

CLASS 9 & 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

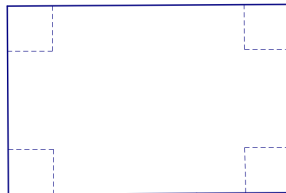
11. Consider a cube with edges of length s . In simplest terms, what is the ratio of the number of cubic inches in the volume of a cube to the number of square inches in its surface area?

(A) $\frac{s}{6}$ (B) $\frac{s}{2}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$ [11] _____

12. Minerva's school library charges a fine on each overdue book. The fine is \$0.25 plus \$0.12 per day. Minerva was fined \$1.09. How many days overdue was Minerva's book?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9 [12] _____

13. Square corners, five units on a side, are removed from a twenty unit by thirty unit rectangular sheet of cardboard. The sides are folded to form an open box. Find the surface area of the exterior of the box, in square units.

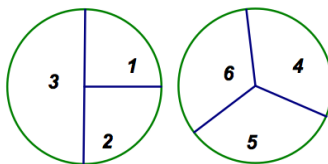


(A) 200 (B) 375 (C) 475 (D) 500 (E) 600 [13] _____

14. Six coins are in a sack. At least one penny, nickel and dime are in that sack. More nickels than dimes, and more dimes than pennies are in the sack. What is the probability of randomly selecting a nickel from the sack?

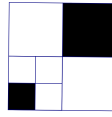
(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$ [14] _____

15. The two wheels shown are spun, and the resulting numbers are added. What is the probability that the sum of the two numbers is even?



(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{5}{12}$ (E) $\frac{4}{9}$ [15] _____

16. What part of the large square shown below is shaded?



- (A) $\frac{1}{8}$ (B) $\frac{2}{7}$ (C) $\frac{7}{24}$ (D) $\frac{5}{16}$ (E) $\frac{5}{12}$ [16] _____
-

17. The data below represent the estimated and the actual costs of raisins in one-half ounce boxes belonging to a group of students. The format is called a back-to-back stem-and-leaf-plot. For example, the top line means that of all the costs, only one estimated cost fell of between 10 and 19, an estimated cost of 16. The second line says that ten estimated costs fell between 20 and 29, three of those being 25. It also says only one actual cost is in this range, 29. What is the median for the actual costs?

Actual costs Ones	Tens	Estimated Costs Ones
	1	6
9	2	0000223555
9999888855422221	3	00015
11100000	4	00005
	5	0000

- (A) 39 (B) 38 (C) 35 (D) 31 (E) 30 [17] _____
-

18. A box containing 40 nails weighs 175 grams. The same box with 20 nails weighs 95 grams. What is the sum of the weight of the box and one nail?

- (A) 4 (B) 9.5 (C) 15 (D) 19 (E) 69.7 [18] _____
-

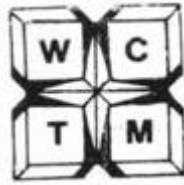
19. After a summer vacation the 1000 students at Gauss Middle School decided to perform a schoolwide mathematical experiment. All students line up single file on the first day of school. When the doors were opened, the first student to enter opened each one of the lockers numbered from 1 to 1000. The second student then closed the even numbered lockers. The third student went to every third locker, shut it if it was open and opened it if it was closed. The fourth student went to every fourth locker, shut it if it was open and opened it if it was closed. This continued until the 1000th student had entered. What is the number of the 7th open locker?

- (A) 13 (B) 17 (C) 49 (D) 64 (E) 81 [19] _____
-

20. Some people got on a bus. At the first stop, two-fifths of those people got off and three-fifths of that original number got on. At the second stop, one-half of the people got off, and one-third of the number left on the bus got on. At the last stop, three-quarters of the people got off the bus, leaving 5 people on the bus. How many people were on the bus before the bus reached its first stop?

- (A) 20 (B) 25 (C) 30 (D) 40 (E) 50 [20] _____
-

Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 3

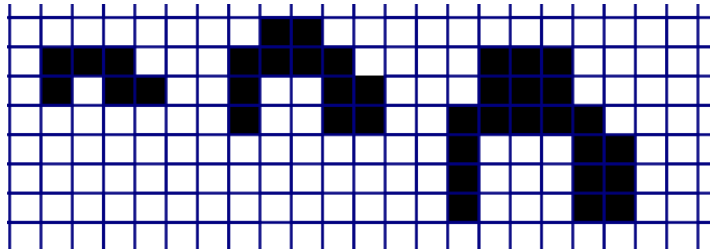
NAME: _____

CLASS 9 & 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

21. Consider the tiling pattern shown below. Find an expression for the number of tiles that would be expected in the 25th figure. Figures 1, 2 and 3 are shown.



- (A) 702 (B) 704 (C) 706 (D) 708 (E) 710 [21] _____

22. Half of the people in a room left. One third of those remaining started to dance. There were 12 people who were not dancing. The original number of people in the room was

- (A) 24 (B) 30 (C) 36 (D) 42 (E) 72 [22] _____

23. The scores on Test D and Test E are shown below. Describe the correlation between the scores on Test D and Test E.

Student	Test D	Test E
Buddy	303	20
Turner	343	53
Kathy	479	70
Suzanne	599	88

- (A) positive (B) negative (C) no correlation (D) quadratic (E) exponential [23] _____

24. Suppose that $\sqrt{\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \frac{6}{5} \cdot \dots \cdot \frac{a}{b}} = 3$ where $b = a - 1$. What is the value of $a + b$?

- (A) 11 (B) 17 (C) 18 (D) 35 (E) 37 [24] _____

25. A cube of edge 3 centimeters is cut into N smaller cubes, not all the same size. If the edge of each of the smaller cubes is a whole number of centimeters, then what is N ?

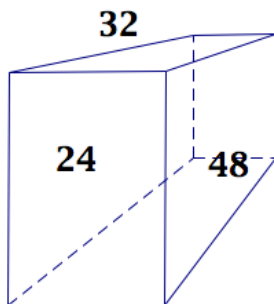
- (A) 4 (B) 8 (C) 12 (D) 16 (E) 20 [25] _____

Go to back \Rightarrow

26. What is the value of x if $4^{20} + 4^{20} = 2^x$?

- (A) 80 (B) 60 (C) 41 (D) 40 (E) 21 [26] _____
-

27. Given a rectangular box as shown below, find its volume in cubic centimeters knowing that the areas of the three different faces of the box are 24, 32 and 48 square centimeters.

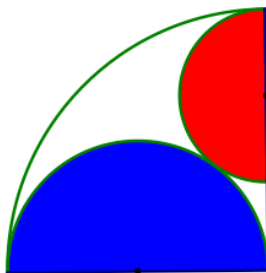


- (A) 104 (B) 192 (C) 208 (D) 576 (E) 36,864 [27] _____
-

28. A gambler sets up a game in which he puts four \$10 bills and one \$20 bill in one box. The person playing against the gambler blindly chooses one of the bills and keeps it. To break even on average, how much must the gambler charge this person to play?

- (A) \$16 (B) \$15 (C) \$14 (D) \$13 (E) \$12 [28] _____
-

29. In the figure below, the two semicircles are tangent. The radius of the quadrant and the diameter of the large semicircle are 2. Find the radius of the small semicircle.

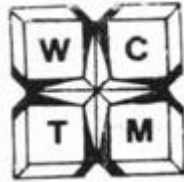


- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$ [29] _____
-

30. Which of the following represents an odd number in base 10?

- (A) 124_{five} (B) 134_{five} (C) 224_{five} (D) 334_{five} (E) 424_{five} [30] _____
-

Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 4

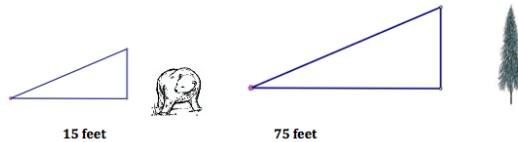
NAME: _____

CLASS 9 & 10 Grade

SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

31. A bear six feet tall casts a shadow fifteen feet long. At the same time, a nearby tree casts a shadow measuring seventy-five feet. How many feet tall is the tree?



- (A) 182.5 (B) 75 (C) 66 (D) 60 (E) 30 [31] _____

32. The following is a partial table of logarithms in the integer base a . Use these values to approximate $\log_a \frac{10}{3}$.

n	2	3	5	7
$\log_a n$	0.387	0.613	0.898	1.086

- (A) 0.523 (B) 0.672 (C) 0.694 (D) 1.086 (E) 1.204 [32] _____

33. The circumference of the base of a right-circular cylinder can is 12 inches. The height is 5 inches. A spiral stripe is painted on the can; it winds around exactly once as it reaches from the bottom to the top. The stripe reaches the top directly above the spot where it left the bottom. If the stripe is the shortest possible going from corner to corner, what is its length, in inches?

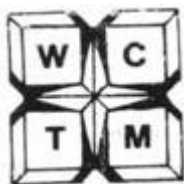
- (A) $5\sqrt{2}$ (B) $\sqrt{61}$ (C) 12 (D) 13 (E) 17 [33] _____

34. Four men, one of whom committed a crime, each made statements. If only one statement is true, who is the guilty man?

Painter 	Chef 	Fireman 	Policeman
"The Chef did it."	"The policeman did it."	"I didn't do it."	"The Chef lied when he said I did it."

- (A) Chef (B) Policeman (C) Fireman (D) Painter (E) insufficient information [34] _____

Wolsborn-Drazovich STATE MATHEMATICS 53rd CONTEST, 2009



Test 5

NAME: _____

CLASS 9 & 10 Grade

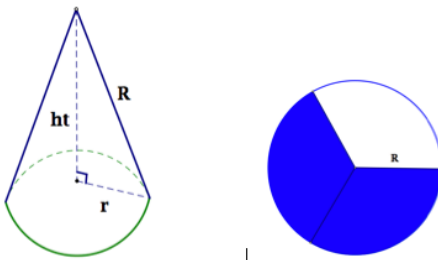
SCHOOL: _____

SCORING: 20 points for each correct answer, -5 for each wrong answer.

41. A safety regulation states that the maximum angle of elevation for a rescue ladder is 72 degrees. If the fire department's longest ladder is 110 feet, what is the approximate maximum safe rescue height, in feet?

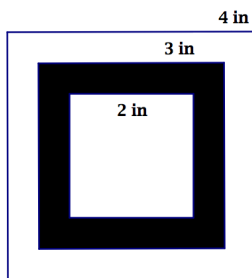
- (A) 27.9 (B) 34.0 (C) 72.0 (D) 104.6 (E) 106.4 [41] _____

42. A circular sheet of paper of radius 6 cm is cut into three equal sectors, and one sector is formed into a cone with no overlap. What is the height, in centimeters, of the cone?



- (A) $4\sqrt{3}$ (B) $4\sqrt{2}$ (C) $2\sqrt{6}$ (D) $2\sqrt{3}$ (E) 2 [42] _____

43. A dart is thrown randomly at the square target shown below. It is made from three concentric squares with side lengths 4 inches, 3 inches and 2 inches. What is the probability that it will hit the shaded region?



- (A) $\frac{1}{3}$ (B) $\frac{3}{4}$ (C) $\frac{5}{11}$ (D) $\frac{5}{16}$ (E) $\frac{9}{16}$ [43] _____

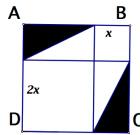
44. If a and b are integers such that $x^2 - x - 1$ is a factor of $ax^3 + bx^2 + 1$, then b is

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2 [44] _____

45. If $(5, 7)$ is a point on the graph of $y = f(x)$ find the coordinates of the corresponding point on the graph of $y = 3 \cdot f(2x + 7) - 5$.

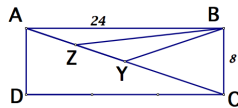
- (A) $(-1, 16)$ (B) $(3, 16)$ (C) $(17, 16)$ (D) $(3, 26)$ (E) $(5, 46)$ [45] _____

46. If $ABCD$ is a unit square, find the area of the shaded region, in square units.



- (A) $\frac{2}{9}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{7}{9}$ [46] _____

47. In rectangle $ABCD$, Y is the midpoint of \overline{AC} , Z is the midpoint of \overline{AY} , $AB = 24$ and $BC = 8$. Find the area of $\triangle ZBY$.

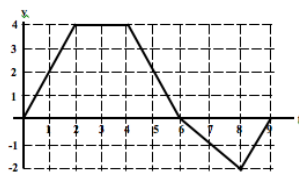


- (A) 12 (B) 16 (C) 18 (D) 24 (E) 32 [47] _____

48. For what value of k will $f(13) = 2009$ if $f(n) = \begin{cases} k & \text{if } n = 0 \\ f(n-1) + 20n & \text{if } n > 1 \end{cases}$?

- (A) 91 (B) 172 (C) 189 (D) 1820 (E) 1918 [48] _____

49. A bug begins to crawl up a vertical wire at time $t = 0$. The velocity of the bug at time t , $0 \leq t \leq 9$, is given by the function whose graph is shown below. At what time t does the bug change direction?



- (A) 2 (B) 4 (C) 6 (D) 8 (E) 9 [49] _____

50. What is the units digit of the decimal representation of 3^{2009} ?

- (A) 1 (B) 3 (C) 6 (D) 7 (E) 9 [50] _____